Evaluation of cost function method to correct flow and scalar fields by combining measured data and CFD

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ABSTRACT

A cost function expressed by the remainders of the governing equations and the difference between observed values and the solutions was proposed. This method was applied for a three-dimensional room and the performances of the cost function method were examined. The individual type solution, which the governing equation of a variable is partially differentiated with respect to itself, and the integrated type solution, which the governing equation of a variable is partially differentiated with respect to another variable, were used and evaluated. The performances of the integrated type solution for combining three data of temperature, concentration and wind components gave the excellent results for all variables than that of the individual type solution.

INTRODUCTION

In fluid dynamic engineering, it is important to understand both flow and scalar fields in a target region. Measurements or Computational fluid dynamics (CFD) are usually used for understanding flow and scalar fields except for simple fields which can be analytically solved and these two techniques had been independently developed(Leonard 1974; Rodi 1976; Launder 1975; Kaga 1993). Since CFD has inevitably errors accompanied by discretization and numerical calculation, CFD can't completely reproduce a complicated field. On the other hand measurements include some errors involving their method. Therefore measured data can't completely satisfy the governing equations and can't cover the whole of a target region because of the difficuluty. In order to understand accurate flow and scalar fields, it is necessary to correct measured data so that the governing equations are satisfied as much as possible and to complement the region without measured data by the governing equations. In this study, a cost function (Shiota 2000) which consists of the remainders of the governing equation and the difference between observed values and the solutions was proposed. This method was applied for a three-dimensional room and the performances of the cost function method were examined.

COST FUNCTION

We defined the cost function (CF) as the sum of the square of two terms that represent the remainders of the governing equation such as the Navier-Stokes equation, the continuity equation and the conservative equations of scalars, and the difference between observed values and the solutions. We introduce an equivalent coefficient for evaluating each term equivalently, an accuracy coefficient based on the accuracy of the observed values, and a weighting coefficient that weights each term according to the purpose of analysis. The cost function is expressed by

$$CF = \int \left\{ \sum_k \alpha_k \beta_k f_k{}^2 \left(\xi_i , \eta_j\right) + \sum_j \alpha_j \beta_j C_j \left(\eta_j - \eta_{j,obs}\right)^2 \right\} d\xi \tag{1}$$

where ξ is the independent variable, η is the dependent variable, f is the governing equation. α, β, and C are a weighting coefficient, an equivalent coefficient, and an accuracy coefficient, respectively. $\eta_{,obs}$ is the observed value. The meaning of each subscript is summarized in Table 1.

Table 1. The meaning of each subscript

	i	j	k
1	coordinate of x direction	wind component of x direction (u)	N-S equation of u component
2	coordinate of y direction	wind component of y direction (v)	N-S equation of v component
3	coordinate of z direction	wind component of z direction (w)	N-Sequation of w component
4	time	Pressure (p)	continuity equation
5		Temperature (T)	conservative equations of temperature
6		Concentration (c)	conservative equations of concentration

The equivalent coefficients are chosen so that each term of the cost function may become equal, when each independent variable changes to the surroundings of the solution by the uncertainty of the same order. However, as the solution is generally unknown, we use the CFD results as an alternative value and vary the values at each point randomly with the maximum errors estimated by assuming the use of typical measurement instruments and measurement techniques. In this study, both a weighting coefficient and an accuracy coefficient are assumed to be a constant of a unity.

The optimum solution is obtained by minimizing the cost function and is expressed by

$$\int\left\{\sum_k \alpha_k \beta_k f_k \frac{\partial(f_k)}{\partial \eta_j} + \alpha_j \beta_j C_j\left(\eta_j - \eta_{j,obs}\right)\right\} d\xi = 0 \tag{2}$$

The combination of $\partial(f_k)/\partial\eta_j$ in the second term of the equation (2) is shown in Table 2. In order to optimize wind component, the Navier-Stokes equation and the continuity equation are partially differentiated with respect to wind component like $\partial f_1/\partial u, \partial f_2/\partial v, \partial f_3/\partial w$ and $\partial f_4/\partial u, \partial f_4/\partial v, \partial f_4/\partial w$. Similarly in order to optimize temperature T, the conservative equation of temperature are partially differentiated with respect to temperature T like $\partial f_5/\partial T$. We call the solution of these types the individual type solution. The conservative equations of temperature and concentration are the function of wind components so that they may be optimized by partially differentiated with respect to wind components u,v,w like $\partial f_5/\partial u, \partial f_5/\partial v, \partial f_5/\partial w$ and $\partial f_6/\partial u, \partial f_6/\partial v, \partial f_6/\partial w$. Similarly the Navier-Stokes equation of wind component w (Boussinesq approximate) is the function of temperature so that it may be optimized by partially differentiated with respect to temperature T like $\partial f_3/\partial T$. We call the solution of these types the intergraded type solution.

Table 2. The combination of $\partial f_k/\partial\eta_j$

f_k

		1	2	3	4	5	6
η_j	u	O	(O*)	(O*)	O	O*	O*
	v	(O*)	O	(O*)	O	O*	O*
	w	(O*)	(O*)	O	O	O*	O*
	p	O	O	O			
	T			(O*)		O	
	c						O

APPLICATION CF

Objective room

The objective region is a three dimensional room as shown in Figure 1. The obstacle is set in the center of this room. Heat source and pollutant source are set above this obstacle. The size of this room is 4000mm (W) × 4100mm(D) × 2200mm (H).

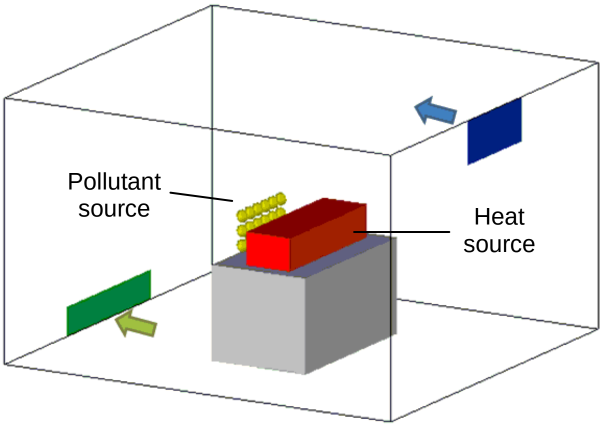


Figure 1. Objective room

Observed value

The observed value can't be measured because of the imaginary room. CFD calculation was carried out according to the boundary conditions as shown in Table 3 and the calculated results were used as the alternative values of observed values. These observed values of wind field, temperature, and concentration in A-A' cross section and in B-B' cross section in Figure 2 are shown in Figure 3.

Table 3. Boundary conditions

Inlet	0.8m/s uniformly
Outlet	Free boundary
Heat source	Heat value is nonuniform. Average 400W
Pollutant source	Amount of emergence is nonuniform.
Wall	Adiabatic condition

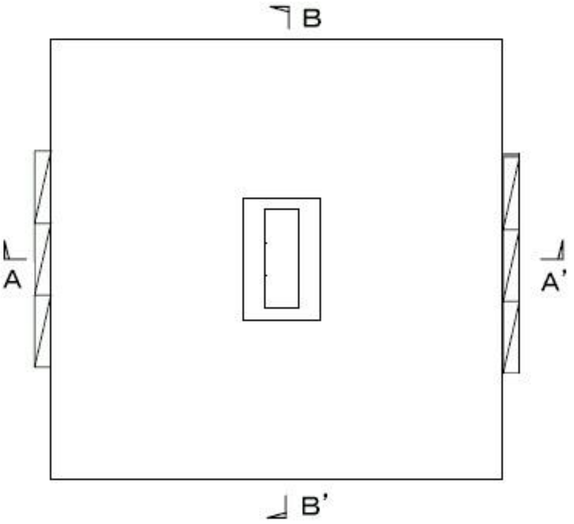
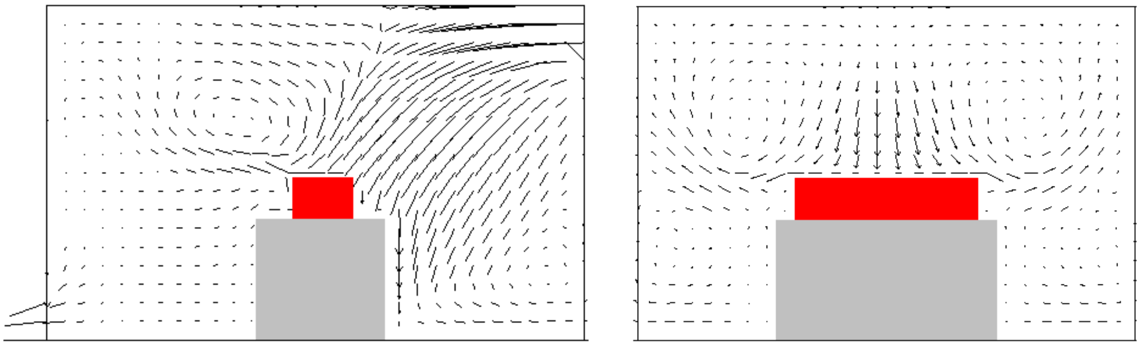
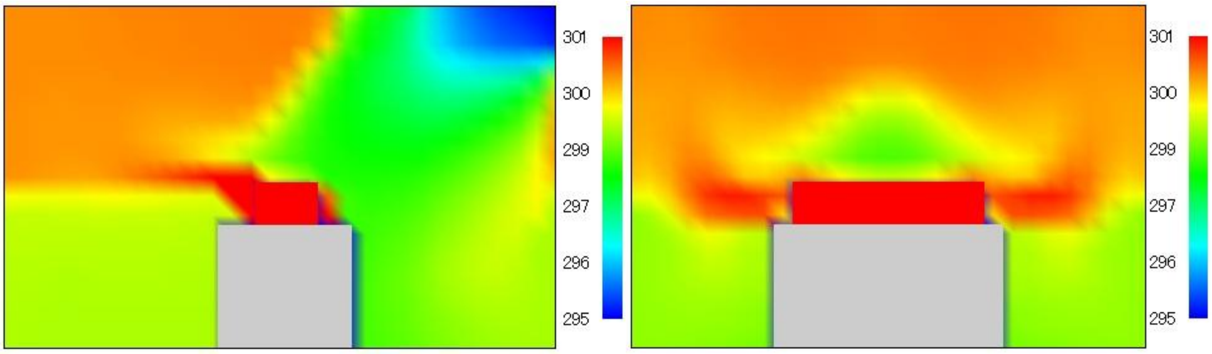


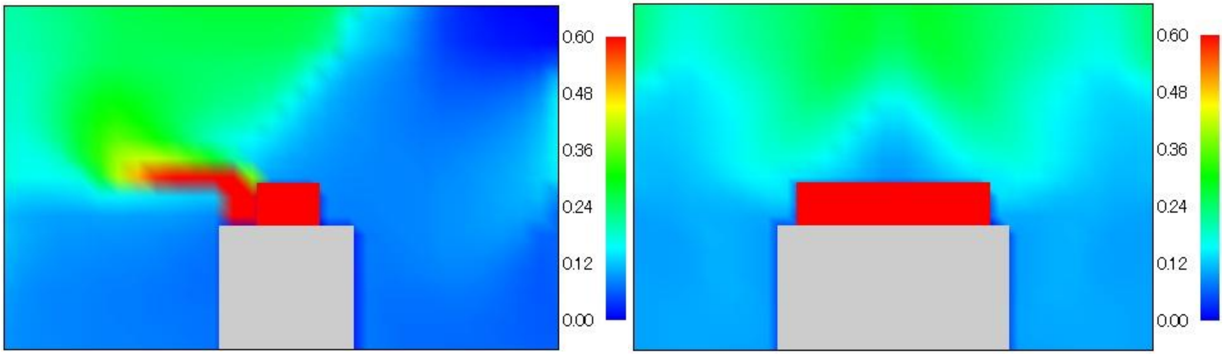
Figure 2. Cross-section



Flow distribution



Temperature distribution



Concentration distribution

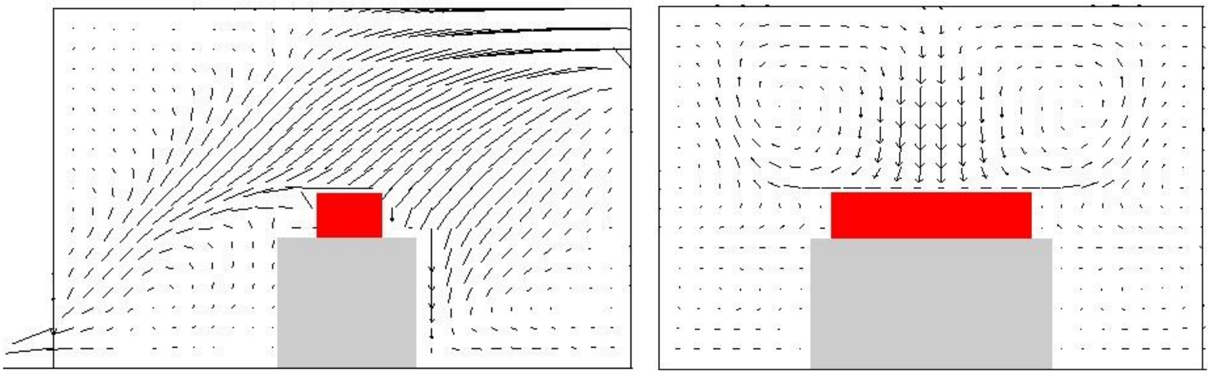
Figure-3. Observed values (A-A' cross-section (left side) and B-B' cross-section (right side))

CFD results

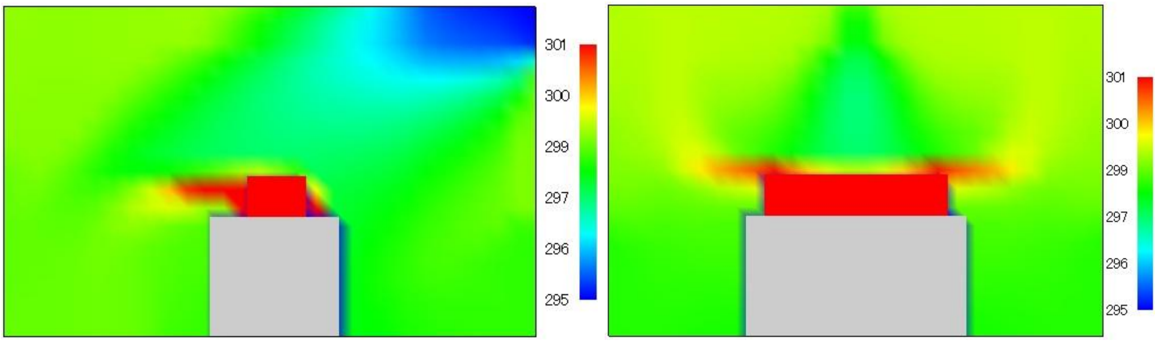
If the boundary conditions of the objective region are obvious, the CFD results become almost same as the observed values in Figure 3. However, the boundary conditions of the actual room are not perfectly obvious. Therefore CFD calculation must be carried out by using the uncertain boundary conditions and the CFD results are somewhat different from the observed values. The CFD calculation by using the incorrect boundary conditions compared with Table 3 was assumed to be the CFD results. The incorrect boundary conditions are shown in Table 4. These CFD results of temperature, concentration and wind field are shown in Figure 4. In Figure-3, the circulating flow occurred above the heat source, but In Figure 4, the circulating flow didn't occur and the downword flow mainly existed. The region with high temperature and high concentration existed above the side of outlet in Figure 3. On the other hand, temperature and concentration distribution are comparatively uniform in the whole room in CFD results.

Table 4. Incorrect boundary conditions

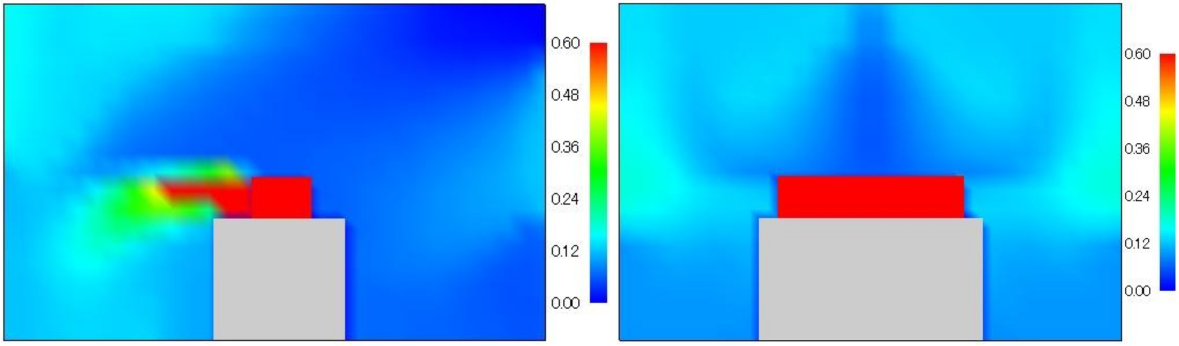
Inlet	0.85m/s uniformly
Outlet	Free boundary
Heat source	400W/m^3: All meshes are uniform.
Pollutant source	All meshes are uniform. Total is the same amount as Table 3
Wall	Adiabatic condition



Flow distribution



Temperature distribution



Concentration distribution

Figure 4. CFD results (A-A' cross-section (left side) and B-B' cross-section (right side))

As the observed values and the CFD results, the flow distributions were greatly different in the outlet side. So, using the statistical index of root mean square error (RMS) (Equation (3)) in the left half side of room where especially the flow distributions is different, RMS between the observed values and the CFD results was summarized in Table 5.

$$RMS(\eta_j) = \sqrt{\frac{\sum_N (\eta_{j,obs} - \eta_j)^2}{N}} \quad (3)$$

Table 5. Error between observed values and CFD results

$RMS(u)$	$RMS(v)$	$RMS(w)$	$RMS(T)$	$RMS(c)$
0.0655	0.0417	0.0534	1.09	0.115

PERFORMANCE OF CF

The CFD results calculated by using the incorrect boundary conditions were corrected by combining the observed values according to the equation (2). As the correct boundary conditions weren't obvious, the governing equations in the equation (2) were solved by using the incorrect boundary conditions.

Individual type solution

The each distribution of CFD results was corrected by using the observed data as shown in Figure 5. Velocity in eight sections and 100 observations of temperature and concentration as shown in Figure 5 were used to correct CFD results.

As shown in figure-6 the velocity distribution in

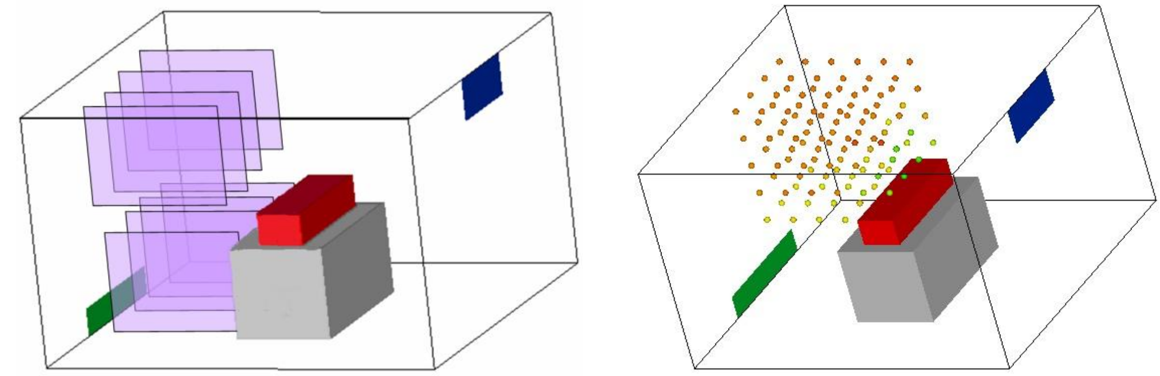
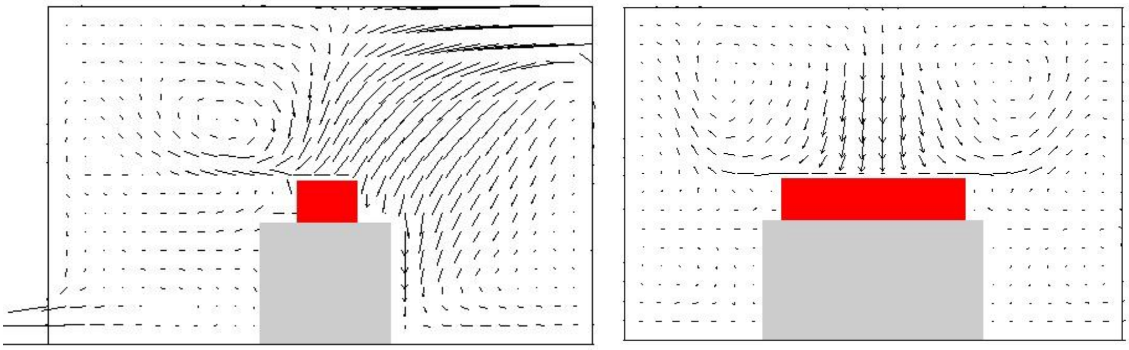
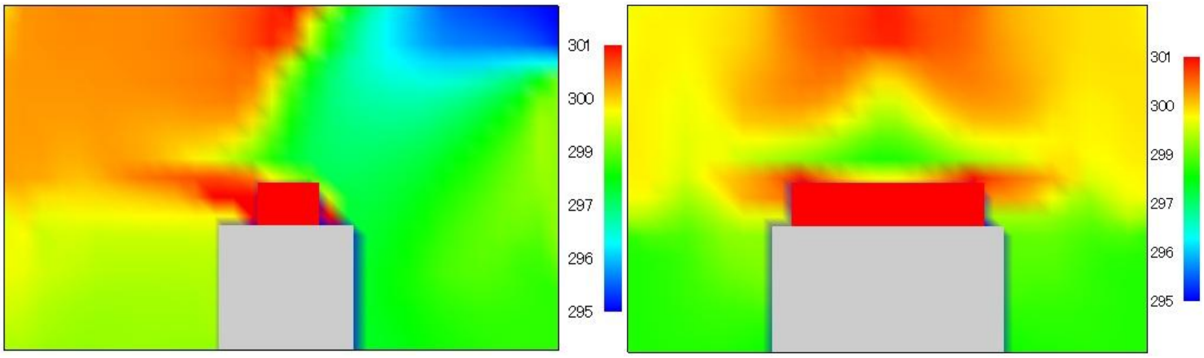


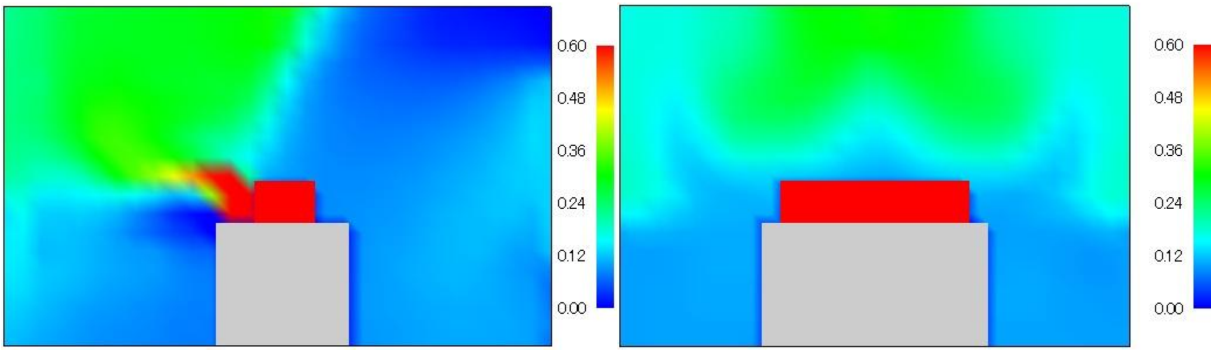
Figure 5. Regions of observed velocity (left side)
and points of observed temperature and concentration(right side)



Flow distribution



Temperature distribution



Concentration distribution

Figure 6. Results of individual type solution (A-A’ cross-sectional (left side) and B-B’ cross-section (right side))

the left side of this room was corrected and the circulating flow was revised. The difuusion of heat and concentration above the room also was revised by the observed values and the corrected velocity.

The performance by individual type solution defined by the equation (4) was summarized in Table 6. This performance was evaluated in the left side of the room as well as Table 5. The individual type solution improved all components.

$$P(\eta)_{RMS} = \frac{RMS_{obs-CFD} - RMS_{obs-revised}}{RMS_{obs-CFD}} \times 100 \qquad (4)$$

Table 6. Performance of CF (%)

	$RMS(\eta)$	$P(\eta)$
u	0.0534	18
v	0.0309	25
w	0.0283	47
T	0.412	62
c	0.0472	59

Integrated type solution

The integrated type solution was applied under the same boundary condition and the same observation values of the individual type solution. The performance evaluated by RMS was summarized in Table 7. This performance was evaluated within region giving temperature observation values of Figure 5. The performances of the integrated type solution for all variables were better than the performances of the individual type solution. Especially, RMS of temperature and of concentration distribution was impressed compared to that of the integrated type solution. These high performances were showed by repeating that the flow fields revised by the scalar data revised the scalar data.

Table 7. Performance of Integrated and Individual type solutions by RMS

	Integrated	Individual	Obs-CFD
$RMS(u)$	0.0468	0.0518	0.0682
$RMS(v)$	0.0311	0.0319	0.0452
$RMS(w)$	0.0327	0.0332	0.0644
$RMS(T)$	0.247	0.387	1.09
$RMS(c)$	0.0413	0.0443	0.120

CONCLUSION

A cost function expressed by the remainders of the governing equations and the difference between observed values and the solutions was proposed in this study. This method was applied for a three-dimensional room and the performances of the cost function method of RMS were examined. In the individual type solution (which the governing equation of a variable is partially differentiated with respect to itself) the performances of the flow fields, the temperature field, and the concentration fields were improved by combining the observed data. In the integrated type solution (which the governing equation of a variable is partially differentiated with respect to another variable) the performances of the flow fields were remarkably improved by combining the temperature data or the concentration data. The performances of the integrated type solution for combining three data of temperature, concentration and wind components gave the excellent results for all variables than that of the individual type solution.

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