Design Programming of Catenary Shell Using Grasshopper Script

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Abstract:

The purpose of this study was to model and design a catenary shell using a modern computer program without performing experiments. The modeling idea stems from the study by Pendergrast (2010), but he listed supplementary items that should be improved in his paper. This study aims to resolve those issues and overcome the drawbacks of the study by Pendergrast (2010). The process of experiment for the design of a catenary shell was reproduced by Grasshopper script. In order to ensure credibility, two models designed from the Grasshopper script were analyzed using a finite element program, SAP2000; one is a square-based catenary shell and the other is a special catenary shell called as the Naturtheater Grötzingen shell (Isler, 1977). First, the developed modeling approach was proved to be reasonable from the analysis of the square-based shell. The reliability was further confirmed by a comparison between the current and previous analysis results for the Naturtheater Grötzingen shell.

Keywords: Spatial Structure, Catenary Shell, Algorithm, Grasshopper, Rhinoceros

1. INTRODUCTION

In order to create large space, spatial structures such as funicular structures with cables, membrane structures, and shell structures are often adopted. Among them, concrete shell structures have more gravity load than other structures, and mostly rely on compressive resistance within the shell due to the characteristics of concrete. Therefore, it is important to properly shape and design the concrete shell such that compressive forces are dominant within the entire concrete shell.

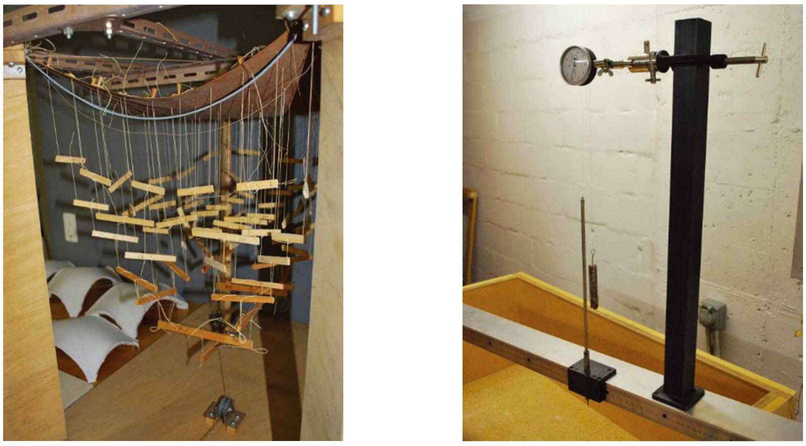
There are several types of concrete shell structures: catenary shell, hemisphere shell and cylinder shaped shell. Unlike hemisphere and cylinder shaped shells, the catenary shell has irregular shape and thus its design is more complicated. The shape of the catenary curve can be found by hanging a heavy uniform flexible cord (e.g., chain) or membrane freely. This is one of the most resistible shapes for gravity load, and the shell of this shape is called as catenary shell.

The catenary shell design has been carried out through the experiment of hanging fabric and the measurement of the curved shape during experiment. This study aims to simplify such a process of catenary shell design, which is expected to contribute to the digitalization of shell design.

2. PRECEDENT STUDY

In the past shell design, the form-finding method developed by Isler (1961) has been popular. This form-finding method is a method which measures the shape of real hanging fabric mixed with mortar, etc. Isler (1961) designed many shell structures using the form-finding method, such as Deitingen Service Station in Switzerland and the Naturtheater Grötzingen in Germany.

Isler' method is one of the basic experimental and empirical methods, and is not compatible with mathematical or computer-based design. As such, this form-finding method cannot be digitalized and the experimental tool shown in Figure 1 is necessary. In Figure 1, the mechanical form-finding rigs are used to simulate uniformly distributed loads. Once the form is determined, the measurement jig is used for accurate measurement of each rig's length. The Isler's method has been used widely before the 21st century, though the experimental process is complicated and inconvenient.



(a) Mechanical Form-Finding Rig (b) Measurement Rig

Figure 1. Tools for form-finding method developed by Isler (Chilton, 2012)

To address the aforementioned issue and digitalize the design process, Pendergrast (2010) developed a design tool replacing Isler's form-finding method by computer simulation. The tool, however, is the stand-alone program that has less compatibility with other computer-aided design programs or finite element packages. The design tool of Pendergrast (2010) is difficult to be applied to actual work-site despite of its significance. Pendergrast (2010) also acknowledged the limitation of applicability and suggested alternatives and other possibilities. One of them is the use of Grasshopper for simulation. The Grasshopper is a graphical algorithm editor integrated with 3D modeling packages in Rhino program (Davidson, 2015).

In this study, an innovative shell design program is developed using the Grasshopper and catenary shells are designed using it. Additionally, the validity is checked by performing finite element analysis for the designed catenary shells and by comparing the analysis results with the previous information.

3. GRASSHOPPER ALGORITHM

Pendergrast (2010) used braced grid wireframes for simulating the behavior of fabric on computer. According to the algorithm of Pendergrast (2010), the gravity force and secondary reaction forces are applied to nodes as shown in Figure 2.

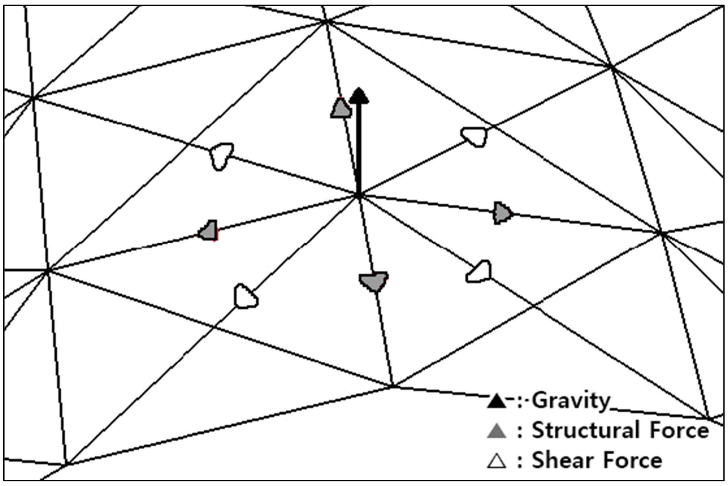


Figure 2. Annotation on portion of wireframe

Each element of wireframes should behave as a spring simulating fabric. Because the Grasshopper has no spring elements, additional plugin is necessary. In this study, the Kangaroo plugin, an interactive physics and constraint solver plugin (Piker, 2010), is used for modeling spring elements. As a result, gravity force and nodal interactions are applicable to the elements.

The Pendergrast algorithm is reproduced in the Grasshopper script. The fact that the wireframe shown in Figure 2 can simulate the behavior of fabric is also mentioned on the Kangaroo manual written by the developer of the plugin (Piker, 2015).

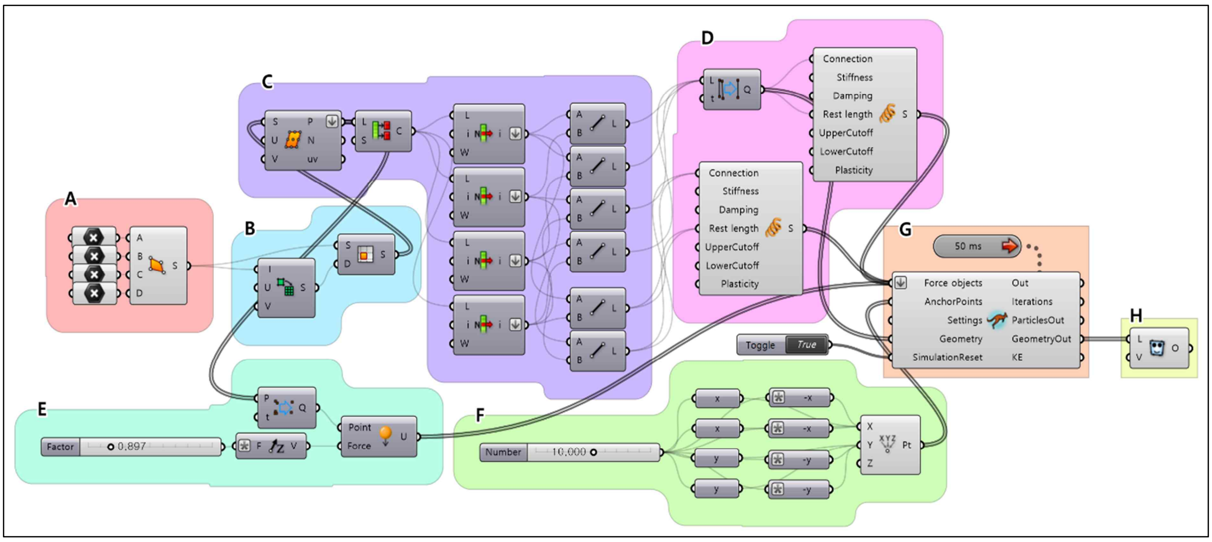


Figure 3. Full grasshopper script for making catenary shell from flat square surface element

Figure 3 shows the full script of Grasshopper, with more detailed step-by-step procedures indicated as follows:

A. Make a surface element from 4 points.

B. Divide the surface element into small surface elements, using Domain function and Isotrim function. All small surface elements have the same size.

C. Find 4 points of each small surface element, and draw grids and braces using the points.

D. Convert the grids and braces into Kangaroo Spring elements.

E. Apply gravity loads to the points.

F. Set Anchor Points and make them able to move using Number Slider.

G. Input the tentative parameters resulting from Step D, E and F into Kangaroo Physics Engine, and execute the simulation of hanging experiment.

H. After stabilization of the shell model shape, make grids and braces to a Mesh object.

In order to establish the wireframe, a flat square surface is divided into smaller elements, and each smaller surface element is converted into spring elements. This conversion is necessary to allow for the elongation of each wireframe member, and in this way the catenary shell can be generated.

The Mesh object from Step H should be exported as a DXF file, edited using AutoCAD, and imported into SAP2000, a finite element analysis program (Abell, 2012). For Step H, the plugin named WeaverBird is used in addition to Kangaroo. The WeaverBird is a topological modeler consisting of existing subdivision and transformation operators (Piacentino, 2009), but it is used only for Step H in this study.

4. COMPARISON WITH PENDERGRAST PROGRAM

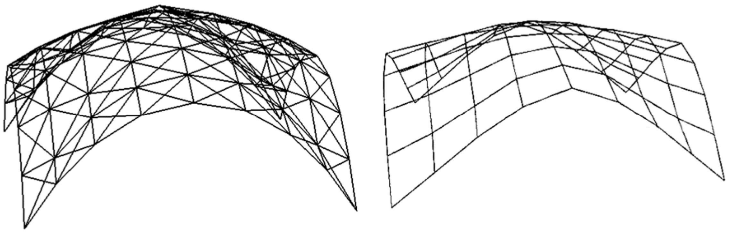
Pendergrast (2010) developed a stand-alone program using C++ and executed a simulation by using this program. Although the Pendergrast program makes it possible to create various user interfaces for imposing loads on points, moving anchor points (only on the x-y plane), etc., this program has some limitations.

Anchor points must exist on the x-y plane in the Pendergrast program. That makes it difficult for the users to generate various shell shapes. Additionally, the output file can only be checked within the Pendergrast program and not exported into any other CAD programs.

On the other hand, the program developed by the authors allows for the movement of each anchor point in the z-direction as well and is compatible with commercially available CAD programs. Furthermore, the developed program utilizes only the built-in and plug-in functions of Grasshopper so that any structural engineers can use with little computational cost, though the user interface can be designed only using built-in basic visual components of Grasshopper such as Slider and Toggle Button.

4.1 Shape Comparison

Figure 4 shows each model from the simulations by the design tool of Pendergrast and Grasshopper script. Both are in the state of force equilibrium. Pendergrast (2010) did not show the stiffness of spring elements, the size of each grid element, and the magnitude of applied loads. Those kinds of parameters are assumed arbitrarily when making flat-square-based shell with the Grasshopper script. Although the differences might exist, the final shapes are similar in general as shown in Figure 4.



(a) by Pendergrast (b) by Grasshopper

Figure 4. Final shell shapes

4.2 Structural Analysis

Pendergrast did not perform the actual structural analysis using the final shell shape, but just performed the form-finding analysis. The comparison between structural analysis results from the design tool of Pendergrast and Grasshopper script cannot be performed on equal terms. In this section, structural analysis is carried out using SAP2000 only for the developed Grasshopper model.

The thickness of shell is set to 250 mm and the span of shell is about 8000 mm. In order to decide whether it is thick or thin shell, Equation (1) is used.

$$\frac{d}{L} = \frac{250\,\text{mm}}{8000\,\text{mm}} = \frac{1}{32} < \frac{1}{20} \tag{1}$$

where d : thickness of shell, L : span of shell

Thick shell behaves like a plate of which the ratio between thickness and span is approximately 1/20 or 1/10. Thus, the ratio of thin shell should be under 1/20. In this case, its ratio is 1/32, so it could be considered as a thin shell. The anchor points have the fixed boundary condition. After the analysis is carried out, the average stress is calculated by Equation (2).

$$f_n = \frac{f}{d} \tag{2}$$

where f_n : value of normal force diagram (kN/m),
f : average in-plane force, d : thickness of shell element

The maximum and minimum values of axial forces are summarized in Table 1. There is no tensile force along the local x axis, while there is moderate tensile force along the local y axis. The shell where tensile force is applied is much smaller than the shell with compressive force. Therefore, this structure is considered as a shell.

Table 1. Minimum and maximum value of axial forces

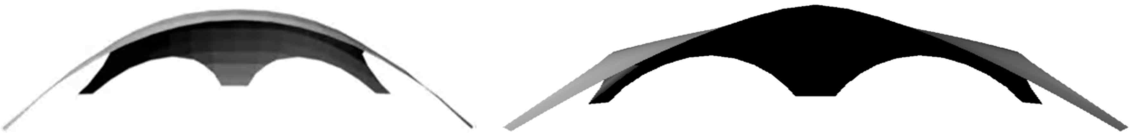
Unit : kN/m	x axis (local)	y axis (local)
Min.	-208.9	-541.2
Max.	152	-19.2

According to the maximum shell stress diagram (not shown), the stress on the shell is about ±0.2 MPa, except for anchor point regions. The maximum stress is 4.54 MPa and the minimum is -3.54 MPa. These values appear near the support anchor points. The absolute values of maximum and minimum are relatively small; thus, a small amount of reinforcing bars can be provided near the anchor points. Overall, the model generated by Grasshopper script is proven to be a catenary shell, which is compression-dominated.

5. COMPARISON WITH PREVIOUS STRUCTURAL ANALYSIS

Maurer (2013) did perform both form-finding analysis and structural analysis of the Naturtheater Grötzingen, a built shell structure in Germany. This structure was designed by the form-finding method of Isler (1961), and Maurer (2013) reproduced the structure through the same method. In this study, the Naturtheater Grötzingen is reproduced using the Grasshopper script and compared to the model of Maurer (2013).

5.1 Shape Comparison



(a) Maurer (b) Grasshopper

Figure 5. Comparison of Naturtheater Grötzingen model by two methods

Referencing the height and span of the shell model analyzed by Maurer (2013), a new model is generated by using Grasshopper, and some unclear parameters are assumed in the analysis. The scale is the same as Maurer's model, with 42 m width by 28 m depth by 10 m height. Figure 5 shows some differences between two models. In the case of the Grasshopper model, the curvature and peak point location are somewhat different, though the overall shape is similar.

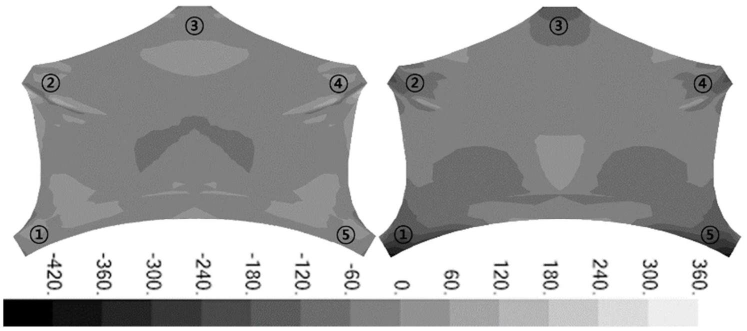
5.2 Structural Analysis

As was done in Maurer's analysis, thin concrete shell elements with a thickness of 105 mm are used. Referring to Equation (1), the ratio between the thickness and span is about 1/420; thus, this shell is considered as a thin shell. The behavior of the shell under self-weight only is analyzed.

The plan of shell is much more complicated than the square model shown in Figure 4, but both models shown in Figure 5 are not an "ideal" catenary shape. For example, some elements in the middle of the shell show a small degree of moment and tensile force. Table 2 shows the analysis result. According to the table, smaller values of compressive and tensile forces/stresses are obtained from the model by Grasshopper. This may indicate that the developed model by Grasshopper is more efficient in distributing gravity loads throughout the shell.

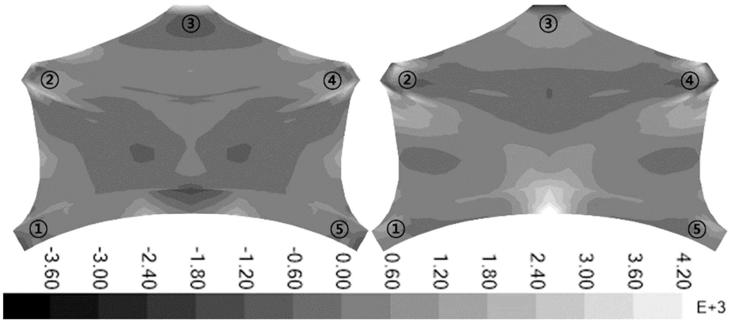
Table 2. Comparison of force and stress of two models

	Grasshopper		Maurer	
	Tens.	Comp.	Tens.	Comp.
Max. Axial Force (kN/m)	393.3	478.1	1152	2785
Max. Stress (MPa)	8.1	3.9	15	14
Min. Stress (MPa)	2.7	10.6	3	33



(a) Local x axis (b) Local y axis

Figure 6. Axial force diagram (kN/m)



(a) Top Surface (b) Bottom Surface

Figure 7. Maximum shell stress diagram (MPa)

In Figure 6, tensile force appears near the support anchor points in the direction of local x axis. Additionally, it occurs near the peak point and along the arch lines between two support anchor points in the direction of local y axis. Despite of the existence of tensile force, the average tensile force is just about 5 to 20 kN/m, while the average of compressive force is about 50 kN/m. The area where compression is applied is larger than the tensile area. The maximum compressive and tensile forces are 478.1 kN/m and 393.3 kN/m, respectively. The analysis results in this section indicate that the shell is in the compression-dominant state. On the other hand, both the maximum compressive and tensile forces in the analysis of Maurer's model are over twice those in the developed Grasshopper model (5.8 times and 2.9 times for compressive and tensile forces).

According to Figure 7, the largest tensile and compressive stresses occur around the anchor points of No. 2 and 4, and along the arch between No. 1 and 5. The maximum tensile stress is 8.1 MPa near No. 2 and 4 anchor points, and the tensile stress along the arch between No. 1 and 5 is increased up to approximately 5.6 MPa. Given that the tensile strength of concrete is about 2 MPa, steel reinforcement at this region is essential.

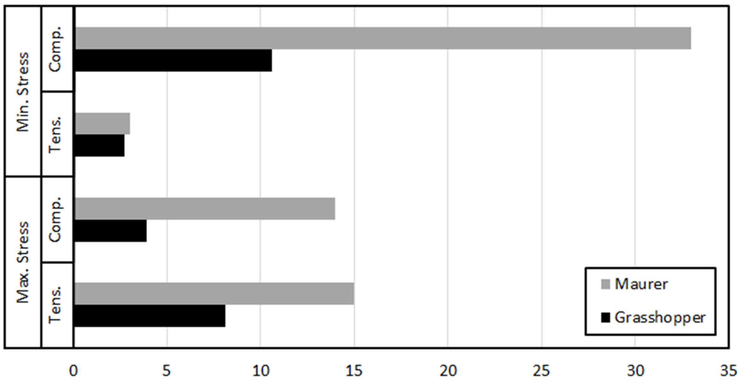


Figure 8. Comparison of stresses between Maurer and Grasshopper models (MPa)

As shown in Figure 8, there are substantial differences of maximum and minimum stresses, except for the minimum value of tensile stress. The values from the Grasshopper model are smaller than the model by Maurer in all cases. The largest difference between Maurer's model and the developed Grasshopper model is the minimum compressive stress (3.5 times difference).

6. CONCLUSION

The reproduction model of the Naturtheater Grötzingen by using Grasshopper algorithm and Pendergrast's form-finding approach is similar to the previous Maurer's model that utilized Isler's form-finding method in terms of the shell shape and stress distribution pattern. Some differences in analysis results include the location of peak point, the existence of double curvature, and the magnitude of absolute stress values. Maurer (2013) mentioned that his model does not reflect the originally built shell perfectly. For example, the Naturtheater Grötzingen has double curvature at the arch parts between anchor points. In Maurer's model, very large shell elements of about 3 m height by 3 m width were used. Conversely, the shell elements used in the Grasshopper model are about 1 m height by 1 m width such that the double curvature is reproduced in the Grasshopper model. The fact that the double curvature is simulated and the maximum and minimum stress values of the Grasshopper model are smaller proves that the developed model could be more efficient in generating catenary shell. Furthermore, the developed model can use the widely available Grasshopper program with ease, whereas the Maurer's approach requires the manual experimental measurement. In the process of hand measurement, it appeared to cause the model's asymmetry. The developed model, however, created perfectly symmetric shell because it is purely computer-based. Although the Grasshopper program can be used fairly easily, the developed algorithm is original and has its significant contribution to the state-of-the-art in form-finding methods.

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