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MODEL UNCERTAINTY AND DESIGN MARGIN IN SAFETY EVALUATION BASED ON BEST ESTIMATE CODE

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ABSTRACT

Atomic Energy Society of Japan has established a "Standard Method for Safety Evaluation using Best Estimate Code Based on Uncertainty and Scaling Analyses with Statistical Approach." The standard is a deterministic safety evaluation method that employs a stochastic approach and considers uncertainties in evaluating the safety parameter on licensing criteria. We define the uncertainty as bias and randomness. The bias reflects our state-of-knowledge. The randomness reflects the stochastic feature in the accident scenario. A probability model that the safety parameter exceeds the safety criterion is proposed and the relationship of the safety margins and the bias and the random error is discussed. Furthermore, quantitative expression of the confidence level is proposed. It is important to reduce the uncertainty of the best estimate code and the safety evaluation process continually. For this purpose, Bayesian approach as well as information entropy are useful technique to measure the value of information in view of updating the code model. Lastly, an example is given to show the effectiveness of the procedure to be applied to the uncertainty reduction process.

1. INTRODUCTION

Needless to say, a safety evaluation is an important process of design and regulation of nuclear power plants (NPPs). The process is to (1) define a parameter or figure-of-merit (FoM) that is used for the decision on the safety; (2) quantify the parameter in most cases based on a safety analysis computer code; (3) decide whether the parameter lies within an acceptable range. In the process, uncertainties are considered and safety margins are added if necessary. The uncertainty is not exactly known and it exists intrinsically in the process. On the other hand, the safety margin is included intentionally.

Requirement on the calculation programs, models and parameters used for the safety analysis is given in the Regulatory Guide for Reviewing Safety Assessment of Light Water Nuclear Power Reactor Facilities by the Nuclear Safety Commission (NSC, 1990). "*The calculation programs, etc. used for the analysis of a postulated event shall be verified with respect to their applicability. The models and parameters for the analysis shall be specified such that they give a severe result to a reasonable extent in view of the objective of the analysis. If there can be uncertain factors in specifying the parameters, appropriate safety margins shall be taken into account.*" In the accident conditions, it is required that "*the core shall not be damaged considerably, and adequate coolable state of the core shall be maintained*." The ECCS Performance Evaluation Guide states the safety criterion (allowance level) of the calculated peak cladding temperature (PCT) is l,200°C.

A question is how "*the reasonable extent*" and "*appropriate safety margin*" are assured and how we recognize the actual safety margins in the safety evaluation process. Figure 1 shows the relationship of a load and a capacity. In the current example, a PCT is the load and the allowance level of the temperature is the capacity. If we foresee uncertain factors, we will add a bias that is to assure the "*appropriate safety margin*". However, it is hidden because we are not certain of the uncertain factors. Accordingly it is reasonable to regard the bias as an uncertain random variable. The capacity also involves a bias that reflects the conservatism of the safety criterion in the ECCS guide. The biases will explain "*the appropriate safety margins*" and assure the "*severe result to a reasonable extent*." A safety margin is quantified easily by comparing the estimated load and capacity (the safety criterion). However, we see that hidden safety margins exist in the procedure between the realistic and estimated values of load and capacity. The hidden margins are not visible. What we can recognize is the difference of the estimated load and capacity, i.e. a visible safety margin.

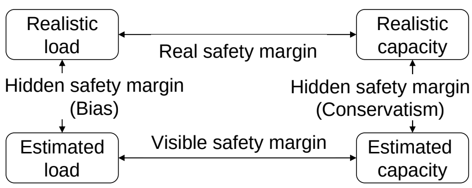

Fig. 1 Realistic and estimated load and capacity.

Since the Tyuetu-Oki earthquake in July 2007, seven NPP units in the Kashiwazaki-Kariwa site have ceased the operation. The acceleration level on the base mat is 2.5 times at maximum of the S2 earthquake that is the extreme design earthquake. There were no loss of function in the safety-grade components and the reactors were successfully shutdown, cooled-down with the boundary and containment integrity in all the units. Why the safety system did not fail even though the earthquake level exceeded the S2 earthquake? Answer is that there are hidden safety margins. However no one exactly knows how much the safety margins are in the current design practice; where the margins come from; and what the appropriate margins should be. These questions are common to the thermal-hydraulic safety evaluation.

Now we need a reasonable and practical methodology and data to predict the realistic load, a procedure to compare the estimated load with its allowance level of the capacity, and a methodology to reveal the hidden safety margins. We have two methods to predict the realistic load in the actual system. One is an experimental simulation based on a simplified and scaled experimental model and the other is a numerical simulation based on approximated mathematical models. In any cases, we must compromise the approximation and/or simplification. Experiments have a scaling issue and the numerical simulations raise verification and validation (V&V) problem. The accuracy and our confidence level on the prediction depend on our knowledge and the simulation capability of the experimental or numerical simulation model. We often interpret the uncertainty and margin as such explain the difference of the model prediction and the actual response that we do not know in advance. The observation in the NPP is a “real world”. There is no approximation and simplification in the real world although variability may exist. On the other hand, models can only approximate or predict the real world but the variability does not appear as far as it is deterministic. The prediction cannot be free from the uncertainty. Hence the uncertainty should be transparent.

2. STATISTICAL SAFETY EVALUATION METHOD

The Atomic Energy Society of Japan (AESJ) has prepared a standard for the statistical method of the safety evaluation of NPPs (AESJ, 2008) and is to be published soon. The approach is hereafter named “the statistical safety evaluation (SSE) method”. In the documentation of the AESJ standard, what is discussed with intense interests is the definition of the uncertainty and the margin. In this paper, the author presents his personal view on the uncertainty and the margins in the SSE. The method described in the standard is based on Code Scaling, Applicability and Uncertainty (CSAU) (Boyack, et al., 1989) and Evaluation Model Development and Assessment Process (EMDAP) (USNRC, 2005). CSAU method has its basis on safety research of a loss-of-coolant-accident (LOCA). The AESJ standard aims at wider application to the safety evaluation such as anticipated operational occurrences and accidents. Requisites for the best estimate (BE) codes employed in the safety evaluation are specified in the EMDAP.

Table 1. Comparison of SSE, DSE and PSA approaches

	Scenario	Assumption	Code	Input	Model	Output
Probabilistic Safety Assessment (PSA)	Comprehensive and probabilistic	Nominal with all possibilities	Best estimate + Systems analysis	Probabilistic & nominal	Nominal with uncertainty	Most probable with uncertainty,CDF, etc
Statistical Safety Evaluation (SSE)	A priori and selective	Deterministic and conservative	Best estimate	Probabilistic & nominal or biased	Nominal or biased	Best estimate with uncertainty range
Deterministic Safety Evaluation (DSE)	A priori and selective	Deterministic and conservative	Conservative	Deterministic & conservative	Conservative	conservative and deterministic

Taking advantage of the recent advance in the computer technologies and numerical techniques in reactor safety analysis, one has been developed so-called BE codes for the plant dynamics with the recent neutronics and thermal-hydraulics models. Establishment of the scaling theory and experimental database for the reactor thermal-hydraulics and safety is another significant momentum of the procedure. Thanking to the progress, it is possible to extrapolate our knowledge in a miniature experiment scale to the actual reactor scale. The standard prescribes the procedure for applying the BE code to the safety evaluation of the plant design and quantifying uncertainties.

In this process, a bounding event, i.e. a design basis event is postulated deterministically. The initial and operational conditions are indefinite but the worst situation is known. Hence the worst and conservative is postulated. We see the scenario and assumptions are treated in the same way as in the deterministic safety evaluation (DSE) as shown in Table 1. In the probabilistic safety analysis (PSA), whole spectrum of accident scenarios are considered and they are treated in a probabilistic manner. Uncertainties in scenarios are evaluated in the framework of the PSA. It is the essential difference between the probabilistic and deterministic approaches. Therefore it can be said that the SSE approach is very similar to the current practice, i.e. the DSE. Concerning the postulated scenario, the safety analysis is performed using the BE code. The analysis is on the nominal basis and uncertainties are considered for input and models.

In the SSE, one selects the Safety Parameter on Licensing Criteria (SPLC) with which we decide whether we accept or reject the hypothesis that a safety design conforms to the safety criterion. A series of uncertainty analyses is performed to obtain the most realistic estimate of the SPLC and its variability with a confidence interval. Confirming the upper bound of the SPLC meets the safety criteria, we quantify the safety margin of the design with regard to the scenario of the plant. The objective of the standard is to rationalize the safety evaluation process. Thus the usage of the BE code and uncertainty estimate is essential in the approach. Advantages of deploying this method are that (1) accountability of the safety evaluation results is enhanced; (2) the latest knowledge would be timely reflected in confirming the plant safety; (3) individual uncertainties pertaining to analytical conditions, analytical codes and modeling are adequately quantified and the uncertainties are transparent; (4) cost-effective safety research programs are proposed and the rationalized plant design can be achieved. In this context, V&V and continual improvement of the BE code and reduction of the uncertainty are very important.

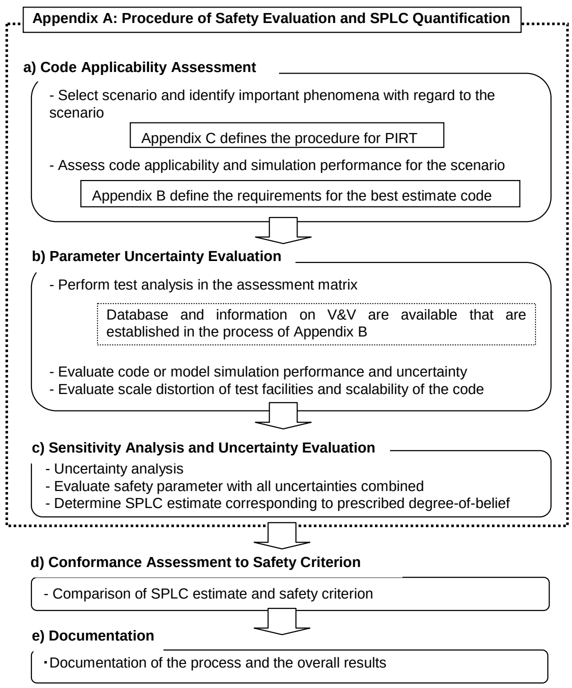

Fig. 2 Outline of the SSE approach.

Figure 2 shows the SSE procedure. The standard consists of the main body, appendices A, B and C. Furthermore, Appendix D gives numerical example and explanatory documents are attached. In the main body five items are defined as shown in Fig.2. The methodology and procedures for SPLC quantification are given in Appendix A. Requirements for BE code and a procedure to establish Phenomena Identification and Ranking Table (PIRT) are given in Appendices B and C, respectively. The SPLC quantification process is further described in detail as a flow chart in Fig. 3. It is notable that four uncertainties are evaluated and biases reflecting each uncertainty are intentionally added to the process. They are uncertainties from (1) separate effect test (SET), (2) integrated effect test (IET), (3) initial and operating conditions and scenario and 4) other sources. The standard defines the uncertainty consists of bias and random variability. The random variability is introduced in the stochastic feature of input data and models. The author recognizes the bias in the SSE, conservatism in DSE and state-of-knowledge in PSA are practically equivalent concept. Discussions on the relationship of the bias and the randomness are given in the following sections.

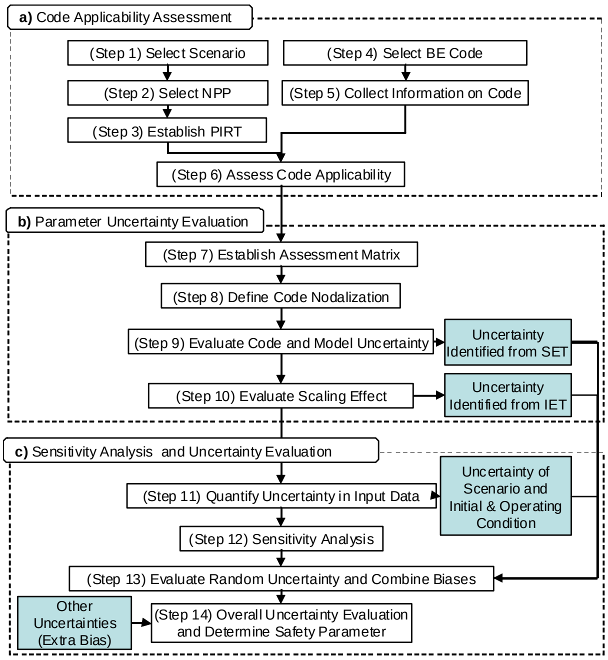

Fig. 3 SPLC quantification process with BE code.

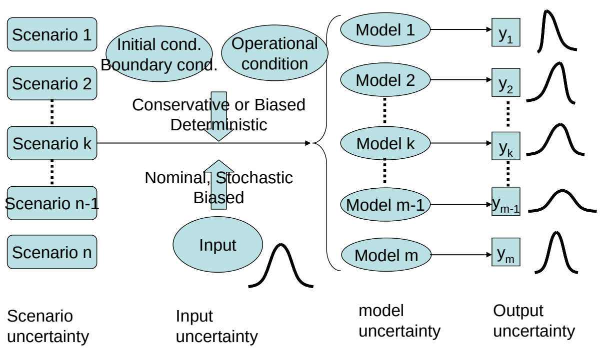

Fig. 4 Uncertainty evaluation using the BE code.

Figure 4 shows the uncertainty evaluation process with the BE code computation. A number of scenarios can be considered for the safety analysis. In the SSE approach as well as the DSE, a scenario enveloping the consequence of similar accident sequences is selected. The input data are treated as random variables and their uncertainties are considered by probability distribution functions (PDFs) of individual parameters. The boundary condition and initial condition are interpreted as a part of scenario and they are decided conservatively and deterministically. The BE code deals with a number of phenomena for each of which one or more phenomenological or mechanistic models (here they are termed sub-models) are used after V&V process. In Fig. 4, a specific combination of the BE code, sub-models and biases in input data and model parameters is called a model. We say the model uncertainty means the bias in the model and input data. Corresponding to the combination of an input data sample and each model candidate, one sample output is generated.

2. DEFINITION OF UNCERTAINTY

2.1 Necessity of Uncertainty Evaluation

We consider the criterion that the PCT does not exceed T_C (=1,200°C) during a large break LOCA. The PCT is evaluated with a BE code. We use the PCT estimate to judge the conformance of PCT to the criterion. However, the physical and numerical models cannot be free from the uncertainty. Hence we are not sure if the estimate is reliable or not and we need to quantify the uncertainty and our confidence level of the PCT estimate. The DSE approach uses conservative code and input data as in Table 1. Physical models are also conservative. Thus we are very confident that the safety requirement is satisfied. The DSE approach assures our confidence by the conservatism. It is important in the reactor safety that we can see how the conservatism is and to what extent of confidence we can rely on in the safety evaluation. The BE and uncertainty analysis is a effective procedure that quantifies those. The confidence level, our degree-of-belief on the models, and the state-of-knowledge are expressed as a probability. Winkler (1993) mentions that since the probability is the language of uncertainty, the uncertainty is expressed with the probability.

The uncertainty problem is discussed based on a statistical test of a hypothesis. Null hypothesis is that PCT exceeds T_C and the safety rule is violated. Since the statistical test can result in one of two outcomes, i.e. rejecting or accepting the null hypothesis. There are two types of errors, i.e. type I error and type II error. Type I error is defined as rejecting the null hypothesis if it is true. Failing to reject the null hypothesis if it is false is a type II error. The probabilities of making these two types of errors measure the risk of making incorrect decisions when we perform a test of hypothesis. In a regulation purpose what is of importance is to prevent the type I error. It is the reason that in the NSC requires the calculation programs shall be verified with respect to their applicability and appropriate safety margins shall be taken into account for the parameters. We want to decide the degree of the conservativeness to be sure not to make the type I error. On the other hand, from the viewpoint of optimal design and economical aspect, the probability of committing the type II error of course should be minimized as well.

The safety evaluation results are used for decision making on safety of the NPP. The evaluation results should be reliable with high confidence level. However, the safety evaluation results include variability and uncertainties. What is the probability of committing type I or type II error? Decision makers who utilize the safety evaluation results need to know whether they can rely on the results or not. The confidence level of the safety evaluation results is an important aspect of quality assurance. The purpose of the uncertainty evaluation is to provide the decision maker with enough information to estimate the probability of making two types of errors quantitatively. Hence the uncertainty and safety margins should be transparent and visible.

2.2 Definition of Uncertainty in SSE Approach

We do not know the real world. What we can see is samples from the real world. Although we are not sure the specific value, we know the FoM lies within an interval. The uncertainty can be expressed using a PDF defined in the interval. If the BE code reflects our best knowledge and experience, the result from the BE code using the nominal input data, nominal initial and boundary condition, and most probable model will give the best approximation of the real world. However, there still exists disagreement between the best approximation and the real world. The disagreement is attributed to uncertainties. The difference can be deviation and/or dispersion. The deviation and dispersion are termed as the bias and random error, respectively. In the SSE approach, biases are introduced to consider the deviation part of the uncertainties.

To compensate the disagreement, biases are added to the best approximation in the SSE approach. The simulation results after the bias addition are the best estimate of the SLPC. The biases consist of four parts as in Fig. 3. One is the bias identified from SET analysis (code and model uncertainty), the second is the bias identified by IET analysis (scaling uncertainty and coupling effect), the third is the bias in initial and operating condition and scenario, and the last is the additional extra bias. Generally the first two biases can be either positive or negative. In the V&V process of the BE code, we compare the code or model prediction with SET and IET. The first and second biases are to compensate the discrepancies in the V&V process. Some phenomena neglected in the model may influence in specific conditions. The reason of the difference may not be explained reasonably, which is the lack of our knowledge concerning the phenomena. Taking the situation into consideration, the biases are introduced in the input data and/or models parameters. All of these explain the difference of the BE code estimate and real world although still random error remains. PDF of the BE code estimate is shown in Fig. 5.

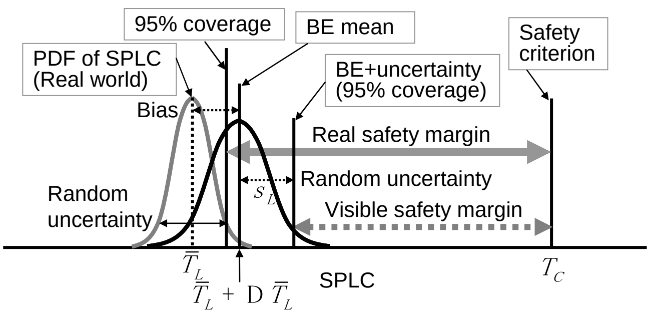

Fig. 5 Uncertainty (bias and randomness) in the SSE.

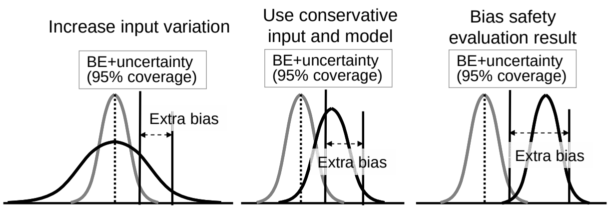

Fig. 6 Definition of extra bias in the SSE.

The starting assumption is that the BE code can predict the worst scenario in the real world exactly with random error. However, the SET and IET may not simulate the real world perfectly. We add extra bias in the step 14 of the SSE approach. It is emphasized that the extra bias is on the basis of engineering judgment. The extra bias compensates the uncertainty in predicting the real world NPP conditions with the BE code that cannot be identified in the SET and IET analyses. This situation could take place when experiment database is not abundant enough to evaluate the standard deviation. Some approximations may cause bias but it is not quantified in the BE code, or interaction effects of multiple parameter uncertainties may cause bias. Also in case that phenomena not modeled in the BE code one cannot estimate the sensitivity or importance of the phenomena and cannot bias the input data. In these situations, extra bias may be added intentionally based on the engineering judgment.

The extra bias could be considered in three different ways as in Fig. 6. One is to increase the variation or randomness of input data. Generally, resultant estimate tends to be broader and the 95% coverage value becomes more conservative. The second approach is to use conservative input data or models rather than nominal and most probable ones. The resultant estimate will become larger. The last one is to add bias directly to the safety evaluation results. The method may not be practical because we cannot justify the reasonableness of the extra bias. It is again emphasized that this addition of the extra bias is exceptional and is performed only to make sure that we do not commit Type I error based on the engineering judgment. The extra bias does not have the same meaning as the other biases which are based on the investigation of the experimental or numerical observations. The author recognizes the criticism that the extra bias may cause "extra conservatism" and is not consistent with the rationalizing approach that we are looking at. However, expert (engineering) judgment is important. The expert judgment has been used in many situations and has been appropriate except very rare exceptions.

Let us consider the PCT as the FoM. We can define the PDF of the tolerance temperature, i.e. the capacity, which is decided based on a systematic series of experiments. Considering the imperfectness of the scaling and simulation performance of the experiments, the tolerance temperature tends to be conservatively estimated. What we observe in the experiments are samples from the parent population. Thus the safety criterion is defined conservatively and deterministically considering the uncertainty. The stochastic feature is considered only in the PCT evaluation process in the SEM method. The same safety criterion as the current DEM approach is used in the SEM.

The resultant of the SSE is expressed as a PDF. 95% confidence level is frequently used. Using the standard deviation, we define the 95% coverage value below which 95% of the samples are to be included. When we compare the 95% coverage of the load with the safety criterion, we recognize the difference is the safety margin that is visible. We understand the visible safety margins are transparent. However, the real safety margin is still hidden because the real load and capacity are not known. The issue of appropriate confidence level, the relation of the confidence level, safety margins and uncertainty will be discussed in section 3.

2.3 Other Definitions of Uncertainty

The uncertainty is discussed in relation to the system modeling in USNRC Regulatory Guide 1.174 (USNRC, 2002) as such "*there are two facets to uncertainty that, because of their natures, must be treated differently when creating models of complex systems." It is common to term aleatory uncertainty and epistemic uncertainty*." According to RG. 1.174, "*The aleatory uncertainty is that addressed when the events or phenomena being modeled are characterized as occurring in a "random" or "stochastic" manner, and probabilistic models are adopted to describe their occurrences. It is this aspect of uncertainty that gives PRA the probabilistic part of its name. The epistemic uncertainty is that associated with the analyst's* *confidence in the predictions of the PRA model itself, and it reflects the analyst's assessment of how well the PRA model represents the actual system being modeled. This has been referred to as state-of-knowledge uncertainty*." Further, the epistemic uncertainty is classified into three categories: *parameter uncertainty*, *model uncertainty* and *completeness uncertainty*.

Another discussion is given in the International Standard Organization for Standardization (ISO,1995) Guide to the Expression of Uncertainty in Measurement. The ISO Guide describes that "*the uncertainty evaluated from statistical analysis of a series of observation and based on frequency distributions is referred to as Type A. The uncertainty determined by judgment and based on a priori distributions is referred to as Type B. In both case the distributions are models that are used to represent the state of knowledge.*" The reason that the two type is considered is "*The purpose of the Type A and Type B classification is to indicate the two different ways of evaluating uncertainty components and is for convenience of discussion only. The classification is not meant to indicate that there is any difference in the nature of the components resulting from the two types of evaluation. Both types of evaluation are based on probability distributions, and the uncertainty components resulting from either type are quantified by variances or standard deviations.*"

The uncertainty definition in the SSE, RG1.174 and ISO Guide are analogous. All of them consider the uncertainties reflect our state-of-knowledge. The SSE and RG 1.174 decompose the uncertainty into lack of knowledge and the random error. The SSE termed the uncertainty related to our state-of-knowledge as bias. The RG1.174 calls epistemic uncertainty that is further classified into parameter uncertainty, model uncertainty and completeness uncertainty. The randomness is designated as the random error in SSE and aleatory uncertainty in RG1.174. The uncertainties are not further classified in the ISO Guide that deals with the measurement uncertainty. Although ease of mathematical handling is emphasized, the uncertainty originated from the state-of-knowledge. It seems the basic idea and concept are common and the uncertainty is expressed by probability.

3. UNCERTAINTY OF LOAD-CAPACITY MODEL

Let L and C be the load and capacity and their random errors are expressed using PDFs. The probability that the load exceeds the capacity is evaluated in the load-capacity model which concept is shown in Fig. 7. In this situation, the load is the PCT, T_L and the capacity is the safety criterion, T_C. We evaluate the exceedance probability that the T_L estimate violates the safety criterion. For simplicity, a normal distribution is used to describe uncertainties of the T_L and T_C and it can be extended to other distributions. Designating the mean and standard deviation of T_L as $\bar{T}_L$ and σ_L, and those of T_C are $\bar{T}_C$ and σ_C, respectively, we write the variables as: $T_L \sim N(\bar{T}_L, \sigma_L^2)$ and $T_C \sim N(\bar{T}_C, \sigma_C^2)$, respectively. The difference of two variables that follow the normal distribution is also normal. Accordingly, we obtain:

$$T_L - T_C \sim N(\bar{T}_L - \bar{T}_C, \sigma_L^2 + \sigma_C^2) \quad (1)$$

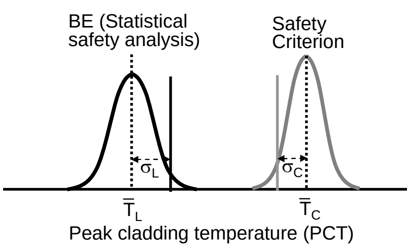

Fig. 7 Concept of load-capacity model.

3.1 Load-Capacity Model (No Bias)

If no bias exists, dispersion around the mean value is considered as shown in Fig. 7. The probability of violating the safety criterion is:

$$\Pr[T_L > T_C] = \Phi\left(\frac{\bar{T}_L - \bar{T}_C}{\sqrt{\sigma_L^2 + \sigma_C^2}}\right) = \Phi\left(\frac{\bar{T}_L - \bar{T}_C}{\sigma}\right) \quad (2)$$

where $\Phi(\cdot)$ is a cumulative standard normal function and the combined variance is $\sigma^2 = \sigma_L^2 + \sigma_C^2$ that expresses the coupled dispersions of the load and the capacity. The dispersion around the mean is expressed by the standard deviation. To maintain the violating probability $P_V = \Pr[T_L > T_C]$ low enough, the PCT should satisfy:

$$\bar{T}_L \le \bar{T}_C + \sigma\Phi^{-1}(P_V) \quad (3)$$

It is regarded that the dispersion reflects both the uncertain bias and random error in Eq. (2). This seems to be the basic idea of the definition of uncertainty in ISO Guide (1995) that deals with the uncertainty of measurement not of modeling. The total error is the point of concern and all the uncertainties are involved in the composit standard deviation σ.

3.2 Load-Capacity Model for Biases Parameter

When we consider the means of load and capacity are biased, the picture looks like Fig. 5. The biases reflect the state-of-knowledge of load and capacity and is expressed as $\Delta\bar{T}_L$ and $\Delta\bar{T}_C$. The bias is uncertain and expressed in a probabilistic manner using a normal distribution as $\Delta\bar{T}_L \sim N(\Delta T_L, \sigma_L^2)$ and $\Delta\bar{T}_C \sim N(\Delta T_C, \sigma_C^2)$ for the load and the capacity, respectively. When we have confidence P_L that T_L does not exceed $\bar{T}_L + \Delta\bar{T}_L$, and P_C that T_C exceeds $\bar{T}_C - \Delta\bar{T}_C$, the probability that the PCT violates the safety criterion is:

$$\Pr[T_L > T_C] = \Phi\left(\frac{\bar{T}_L + \Delta\bar{T}_L - (\bar{T}_C - \Delta\bar{T}_C)}{\sigma}\right) \quad (4)$$

where σ is a random error. Using $\Delta\bar{T}_L = \Delta T_L + \sigma_L\Phi^{-1}(P_L)$ etc., $\bar{T}_L$ should satisfy the following equation to assure P_V is small enough,

$$\bar{T}_L \le \bar{T}_C - \Delta T_L - \Delta T_C + \sigma\Phi^{-1}(P_V) - \sigma_L\Phi^{-1}(P_L) + \sigma_C\Phi^{-1}(P_C) \quad (5)$$

We see the bias and random error correspond to the epistemic uncertainty and aleatory uncertainty, respectively, in RG 1.174. We consider the dissipation is divided into three parts, bias on the load $\sigma_L\Phi^{-1}(P_L)$, bias on the capacity $\sigma_C\Phi^{-1}(P_C)$ and the random error $\sigma\Phi^{-1}(P_V)$. The same holds for the confidence level. Eq. (4) for various confidence levels are shown in Fig. 8. The median curve assumes $P_C = P_L = 0.5$. The 95% upper bound (UB) curve, $P_C = 0.95$ and $P_L = 0.05$ while the 5% lower bound (LB) curve $P_C = 0.05$ and $P_L = 0.95$. The PDF of $\bar{T}_L - \bar{T}_C$ are also shown in Fig. 8. The 5% level corresponds to $P_V = 0.5$ of the 5% LB curve and 95% level corresponds to $P_V = 0.5$ of the 95% UB curve. The violating probability P_V is read from the vertical axis. The circle in bottom left of Fig. 8 is the design point when we require 5% violating probability with 95% confidence on the combined load and capacity.

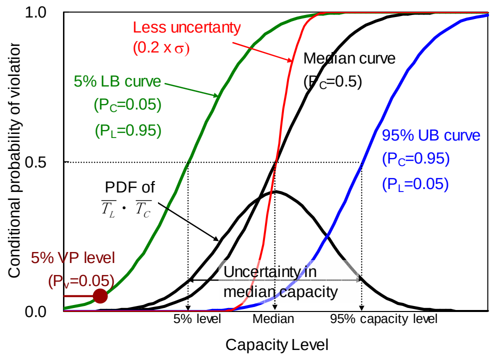

Fig. 8 Comparison of SPLC estimated and safety criterion.

From another side view, we interpret the right hand side of Eq. (5) indicates the safety margins. The fourth term expresses the margin for the random error with confidence level P_V. The fifth term is the margin concerning the bias of the PCT estimate with confidence level P_L. The sixth term is the margin concerning the bias of the safety criterion with confidence level P_C. The slope of the violating probability curve corresponds to the magnitude of the random error $\sigma = \sqrt{\sigma_L^2 + \sigma_C^2}$. If the random error is reduced to 20% of the original value, the curve becomes steeper as shown by the red line in Fig. 8. We interpret that σ_L and σ_C are random errors in the PCT estimate and safety capacity, respectively. If $\sigma_C = \sigma_L = 0$ holds, randomness disappears and SPLC is definite. However, the uncertainty of $\bar{T}_L - \bar{T}_C$ (bias) remains.

3.3 SSE and DSE Approaches

In the current practice, a conservative safety criterion is given in the NSC Regulatory Guide. The capacity is given by the point estimate T_C that equals 1,200°C and $\sigma_C = 0$. The uncertainty of the capacity is involved in the safety criterion and is not seen explicitly in both SSE and DSE approaches. In the SSE method, the probability that the PCT exceeds the safety criterion limit is easily evaluated as:

$$\Pr[T_L > T_C] = \Phi\left[\frac{\bar{T}_L + \Delta\bar{T}_L - T_C}{\sigma}\right] \tag{6}$$

The condition of assuring low violating probability is:

$$\bar{T}_L \le T_C - \Delta T_L + \sigma\Phi^{-1}(P_V) - \sigma_L\Phi^{-1}(P_L) \tag{7}$$

The safety margins coming from the random error is $\sigma\Phi^{-1}(P_V)$ and from the bias is $\sigma_L\Phi^{-1}(P_L)$. The confidence levels of the PCT estimate is P_L and of the violating probability is P_V, respectively.

In the DSE approach, the point estimate of the PCT and the capacity is obtained conservatively and is given by T_L and T_C, respectively. In this procedure, the random errors are not explicitly defined, that is $\sigma_L = \sigma_C = 0$. Hence the probability that the PCT exceeds the safety criterion limit is not evaluated. Instead, the conservatism assures safety margins are enough.

3.5 Confidence Level of Sampling

Eq. (5) expresses how the uncertainties are related to the safety margins and the confidence level. We have discussed two kinds of uncertainties, the bias and the random error. The bias is attributed to the inadequateness or imperfectness of our knowledge. The standard deviations describing the uncertainty is σ_L for the load (PTC), σ_C for the capacity (safety criterion) and σ for the random error.

There is another uncertainty in the SSE approach. The PDF of the PTC is evaluated from a number of computations. It is a process of sampling from a mother distribution. The sample mean and sample variance are only estimates of those in the mother distribution and they coincide only if the number of samples is sufficiently large. The forth uncertainty is attributed to the number of sample calculations is limited. Test of proportion (Crow et al., 1960) is to classify if the samples succeed or fail, if they are perfect or contain defects. The null hypothesis is that the violating probability is greater than P_V. When we find *r* failures from *n* computations, we reject the null hypothesis with confidence level P_S if

$$1 - \sum_{s=r}^{n} C_s^n P_V^s (1 - P_V)^{n-s} \ge P_S \tag{8}$$

holds. Thus the violating probability is assured to be less than P_V at confidence level P_S. Eq. (8) is the well-known Wilks' formula. For example, to assure the violating probability less than 5% with 95% confidence level, we need 59 computations with no violation. The confidence level concerning the sampling size is P_S. Now we summarize that four confidence levels are to be considered; load bias P_L, capacity bias P_C, random error P_V, and sampling size P_S.

We assign the margins with which low probability of violation with high confidence level is achieved. 5% violating probability with 95% confidence level is used as "High Confidence Low Probability of Failure: HCLPF". If we assume $P_C = 0.05$ and $P_L = 0.95$, 5% of violation probability $(P_V < 0.05)$ is assured with 95% confidence level $(P_S = 0.95)$. Taking the 95% confidence level of the load, capacity and randomness into account, we estimate the overall violation probability is $(1\text{-}0.95)^3 = 1.25\text{x}10^{-4}$ with 95% confidence. Because of this low violating probability, it seems reasonable and practical to employ 95% confidence level for P_S and 5% confidence level (violation probability) for P_V. Important conclusions are that the uncertainty and the safety margins become visible.

4. UNCERTAINTY REDUCTION

4.1 PDF of Model Output

The best estimate of the SPLC is obtained by the BE code with biases and stochastic input data. The uncertainty of the SPLC estimate consists of the bias and random error. The bias is related to our state-of-knowledge. Therefore, the uncertainty can be reduced by enhancing our knowledge level. In this section we consider a methodology to reduce the uncertainty (Yamaguchi, 2008) of the SPLC estimate and how newly observed information is utilized in the uncertainty reduction process.

Since the bias is added to the model parameter and input data of the code, it is called a model uncertainty. The model uncertainty is included in the uncertainty evaluation process of the PCT. In the BE code, several sub-models can be used to simulate the same phenomena such as heat transfer, coolant leakage, etc. Let us assume the number of alternative models is *m*. It does not matter if the model means a code or a sub-model. We may select the most probable model, the most conservative model, or mixture of those. If one is confident on the model accuracy, the most probable nominal model can be used. If the model prediction scatters and is uncertain, we may use the conservative model which includes a bias. Our confidence depends on our knowledge on the phenomena and model. The discrepancies between a model and an experiment are to be analyzed and the root cause of the difference is identified, that will improve our state-of-knowledge. It is not appropriate to ask if a model is correct or incorrect. A model is a mathematical expression or approximation of phenomena. We can only say whether we believe the model or not and the degree-of-belief. For some phenomena we may not have a reliable model at all, and for another we have more than one reliable model.

The model output calculated by model *i*, input vector $\mathbf{x}$ and a model parameter vector $\boldsymbol{x}$ is expressed as.

$$y_i = M_i(\mathbf{x}|\boldsymbol{x}) \tag{9}$$

We regard $\boldsymbol{x}$ as the biases for the BE code. If $\mathbf{x}$ and/or $\boldsymbol{x}$ are random, y_i is also random. We write the PDF of y_i as $f_{\mathbf{x},\boldsymbol{x}}(y_i)$. Likewise, we can define $f_{\mathbf{x}}(y_i)$ as the PDF of y_i when $\mathbf{x}$ alone is random and $f_{\boldsymbol{x}}(y_i)$ when $\boldsymbol{x}$ alone is random. $f_{\mathbf{x}}(y_i)$ corresponds to the random error of the PCT calculated from the BE code with stochastic input data since $\mathbf{x}$ reflects the stochastic feature of the input data. $f_{\boldsymbol{x}}(y_i)$ reflects the uncertainty in the model parameter, that is the bias. Although one model is considered in Eq. (9), several alternative models can be used at the same time if appropriate. In this situation, results obtained from individual models are "belief-averaged". If we have a degree-of-belief b_i on the model *i*, and *m* models in total are used, the expected value of y in terms of the degree-of-belief is expressed as:

$$\bar{y} = \sum_{i=1}^{m} y_i b_i \Big/ \sum_{i=1}^{m} b_i \qquad (10)$$

4.2 Bayesian Update of Bias

Our assessment is based on our knowledge and degree-of-belief. If we have new affirmative evidence, we would have more confidence on the prior assessment. The affirmative means the observation y is close to the maximum likelihood estimate as expected in advance. On the other hand, our prediction will be more ambiguous when unexpected, sometimes unfavorable, evidence is obtained. If the observation is far from the maximum likelihood we have to change the prior judgment. The more confident we are, the less uncertainty is assigned and the less confidence we have, the more uncertainty is added. Recall that the bias or epistemic uncertainty reflects our state-of-knowledge or degree-of-belief.

Advance in our knowledge is a result of new information that can be used to update our judgment. The author believes the Bayesian method can reasonably update our degree-of-belief under uncertain situations in a mathematical fashion. We write the prior PDF of a model parameter x as $g(x)$ with mean x and standard deviation s_x. $g(x)$ is updated after we obtain new evidence *E*. The conditional probability that we observe the information being given the model parameter is called the likelihood $L(E|x)$. According to the Bayes theorem, the posterior PDF of x is evaluated as:

$$g(x|E) \propto L(E|x)g(x) \qquad (11)$$

We need to have likelihood function to perform the Bayes inference. Recalling $f_{\mathrm{x},x}(y_i)$ is the probability that y_i is observed on condition that x is given if we select model *i*. The likelihood is the model itself that we believe is likely and that gives the probability of the evidence. Now let us consider x. It is the input data for the BE code or the test condition such as the test pressure and aspect ratio of the vessel, etc. We rewrite Eq. (11) so that the conditions and parameters are explicitly indicated:

$$g(x|E,\mathrm{x}) \propto L(E|x,\mathrm{x})g(x|\mathrm{x}),\quad L(E|x,\mathrm{x}) = f_{\mathrm{x},x}(y|\mathrm{x},x,M) \qquad (12)$$

Here we omit subscript *i* representing a specific model. When we apply Eq. (12) to the current problem mentioned in the previous section, we may regard x as the bias in the model parameter, initial and operating conditions and input data. *E* may be the observed T_L or the fact that T_L conforms the safety criterion.

In summary, we can use Eq. (12) to update our estimate of the bias. We decide models to be tested and the test condition x first of all. Next select an appropriate model or a group of models to establish the likelihood. Then perform a test and obtain evidence. Using the test result (evidence) and the prediction (likelihood), we update the bias. If we use another model, we have a different likelihood and another update. That is to say, a bias is specific to the model or a group of models.

4.3 Value of Information - Entropy

What kind of test output and which test conditions should be selected to update our state-of-knowledge with regard to the bias most efficiently? In other words, how can we measure the value of information? The logarithmic likelihood and the information entropy will give an answer to the questions. A technique is presented to decide the most informative test to reduce existing uncertainties. The value of observing evidence *E* is evaluated by logarithmic likelihood]:

$$V(E|x,\mathrm{x}) = -\ln L(E|x,\mathrm{x}) \qquad (13)$$

The logarithmic likelihood varies from zero to infinity. If the observation is very rare, the likelihood is nearly zero and the logarithmic likelihood is almost infinite. We understand that an occurrence of a rare event is very valuable and we appreciate the evidence. On the other hand, the logarithmic likelihood is zero if the likelihood is unity. We take it granted for that observation of a very probable event is not noteworthy at all. Small logarithmic likelihood implies the observation is meaningless or the information value is negligibly small. The logarithmic likelihood is a measure to judge if evidence already obtained deserves attention or not.

Since we do not know the results in advance, the expected value of the logarithmic likelihood with respect to all the possibilities is a point of concern. Before we perform the test, the expected value is evaluated, that is defined as information entropy:

$$E(x|\mathrm{x}) = \int L(E|x,\mathrm{x}) \ln L(E|x,\mathrm{x})\, dE \qquad (14)$$

The information entropy tells us how the current test condition is considered to be useful to update the bias. In other words, a test is worth being performed if the entropy is large enough. If we plan to perform a test from now, the test condition should be decided so that the entropy becomes the maximum.

As seen from Eqs. (13) and (14), the logarithmic likelihood and entropy are functions of the bias. The value of the additional test depends on the subjective judgment on the prior bias. We have the prior PDF of the bias as in Eq. (12). Thus the logarithmic likelihood and the entropy may be integrated with respect to the PDF of the bias x to obtain the marginal quantities. Expected logarithmic likelihood $\bar{V}(E|\mathrm{x})$ and expected information entropy $\bar{E}(\mathrm{x})$ are given by:

$$\bar{V}(E|\mathrm{x}) = \int V(E|x,\mathrm{x})g(x|\mathrm{x})\,dx \qquad (15)$$

$$\bar{E}(\mathrm{x}) = \int E(x|\mathrm{x})g(x|\mathrm{x})\,dx \qquad (16)$$

The expected information entropy is the value of a test such as a thermal-hydraulic test with condition x before we have the test result for someone who believes $g(x|\mathrm{x})$. The expected logarithmic likelihood is the value of evidence with condition x after we know the test result for somebody who believes $g(x|\mathrm{x})$

4.4 Example – Uncertainty and Entropy

Likewise we assume the bias $\Delta \bar{T}_L : N(T_L, s_L^2)$ in section 3, we consider $x : N(x, s_x^2)$. The uncertainty of x is updated using evidence *E*. We perform a test with parameter vector x. As a result, we obtain the test result y (evidence). The prior PDF of the model parameter (bias) is $g(x|\mathrm{x})$. What is the posterior bias $g(x|E,\mathrm{x})$? Here we have a computer code (model) *M*. The likelihood of the test result is $f_{\mathrm{x},x}(y|\mathrm{x},x,M)$. We can evaluate the posterior by calculating Eq. (12).

There can be various likelihood functions because likelihood is the mathematical form of a model. We consider a criterion. The result is success if $y > y_C$ and otherwise failure.

$$L(E = success|x,\mathrm{x},M) = \Pr[success] = \int_0^{y_C} f_{\mathrm{x},x}(y|x,\mathrm{x},M)\,dy \qquad (17)$$

We have two possible test results, success or failure. The Bayesian method updates the mean and the standard deviation of the model parameter x. The updated x for the two possibilities is shown in Fig. 8 as a function of test condition bias. The biased test condition is written as $\mathrm{x} + x$. If $x = x$, the bias is median-centered and best approximation is expected. If $x > x$, it is over-biased. Also shown is the expected posterior x with regard to all the possibilities (success and failure). It is seen that posterior x increases or decreases according to the test results. The expected posterior x varies a little around the prior x.

Figure 9 shows the reduction of the standard deviation s_x of the model parameter. It is notable that the posterior s_x is smaller than the prior s_x regardless the test conditions and test results. Apparently we have more information after the test than ever. It seems the reduction in the standard deviation reflects the increase of the knowledge. The extent of reduction depends on the test condition and result. The expected posterior standard deviation reaches the minimum when the test condition is nominally biased.

The standard deviation reflects the ambiguity of our knowledge on the bias. The expected entropy is the measure of the value of an attempt to obtain new information for reducing the ambiguity. It is interesting to compare the two quantities. The comparison of the expected entropy and the reduction of s_x is shown in Fig. 10. Relative shapes of the two curves are in good agreement with each other. Value of information is equivalent to the degree of uncertainty reduction if we have the information. From the results, it is concluded that we can design the additional test condition according to the expected entropy.

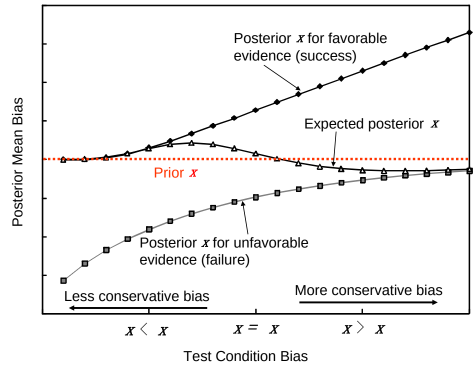

Fig. 8 Bayesian update of mean bias.

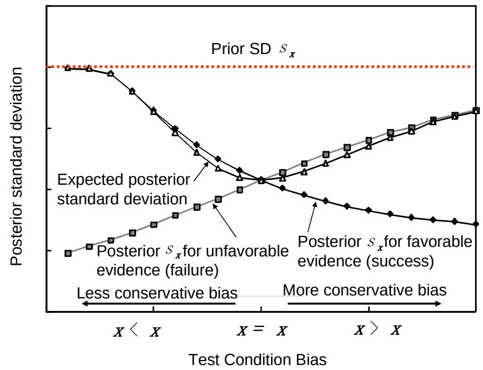

Fig. 9 Bayesian update of bias standard deviation.

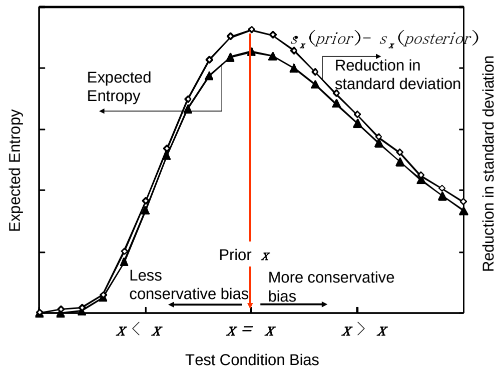

Fig. 10 Uncertainty reduction and entropy.

5. CONCLUSIONS AND FUTURE WORKS

The purpose of the safety evaluation is to make a judgment for the safety regulation of NPPs so that the type I error should be avoided. In the SSE approach, the assumptions reflecting the worst case scenario is used as in the DSE approach. With regard to the input data and models, the best estimate concept is applied and the uncertainties are quantified. Through this process, the probability of committing type I error is quantified and our confidence on our judgment becomes explainable. It is an advantage that the safety margins are explicitly expressed in terms of the bias and the random error. The safety margins are originated from the confidence level regarding the safety criterion P_C, the bias in evaluation process P_L and the random error P_V. Low probability of failure to conform the safety criterion is assured by the confidence level of the sampling error P_S. The overall violation probability is roughly evaluates to be low enough (~10^{-4} with 95% confidence level). It is shown that the Bayesian method is useful to reduce the bias and the information entropy is a effective measure of the uncertainty reduction.

It is important to investigate the root causes of the uncertainties and to reduce the uncertainties of safety evaluation continually, in other words, to enhance the state-of-knowledge. With the present approach, we understand the safety margins involved in the safety evaluation process are visible. In the future, it may be useful to discuss the rationality and the margins of the postulated scenario and the safety criterion in consideration of the state-of-knowledge.

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