Epistemic Uncertainty Reduction in the PSA of Nuclear Power Plant using Bayesian Approach and Information Entropy

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Abstract: It is a great concern how effectively the epistemic uncertainty can be reduced in the probabilistic safety assessment (PSA) as the safety research advances and new knowledge is obtained. The only way to reduce the epistemic uncertainty seems to be to extend our knowledge concerning the uncertain phenomena or parameters. If we have new experimental data, the evidence may suggest improvement or modification in the model and the database for the safety analysis. It is reasonable and effective to collect the seismic fragility data concerning the risk-dominant contributors. On the other hand, if the importance of the information is small, we would take little notice of the data. The issue is how to design the new experiment so that we can effectively improve the model and the database. In the present study, the authors propose a new methodology to measure the reduction in the epistemic uncertainty on the basis of the information entropy concept. The prior and the posterior seismic fragility is updated from experimental data using the Bayesian method. We can determine the experimental conditions that enhance our knowledge on the phenomena in the most effective and economical way. As a result, the uncertainty of the seismic PSA results is reduced on the basis of metrics expressed as the information entropy.

Keywords: Seismic PSA, Information Entropy, Seismic Fragility, Uncertainty

1. INTRODUCTION

Safety assessments of nuclear power plants require the investigation of the occurrence and consequences of rare events. In the regulatory guide for reviewing seismic design of nuclear power plant (NPP) facilities[1], it is stated that we recognize the “residual risk” that an earthquake ground motion goes beyond the design basis earthquake ground motion. According to the guide, the “residual risk” should be minimized if a practical risk-reduction approach is available. Hence it is important to understand the uncertainty of the NPP performance in the rare events.

The residual risk is evaluated using the methodologies written in the standard for the seismic Probabilistic Safety Assessment (PSA) procedure.[2] We have experienced earthquakes that exceed the design earthquake level during these years in Japan. Such an example is the Tyuetsu-Oki earthquake[3] that occurred in July 2007 near Kashiwazaki-Kariwa nuclear power plant island. Seven units are under the influence of the ground motion. The acceleration level on the base mat reached as much as three times of the design basis ground motion in one of the seven units. It is noted that every safety function is successful during and after the earthquake. The NPPs are safely shutdown and cooled down stably. It may suggest the seismic performance of the NPP is more superior than we have expected.

Considering the situations, failure limit or ultimate strength is a point of concern for the safety-grade equipment and systems. It is necessary to quantify and to understand the safety margin that must have been existent. The margins may be defined as the distance between the design point and the failure point as shown in Fig. 1. For a structure such as a beam or cylinder, it is simple to evaluate the structural reliability by comparing the stress and strength. However, it is not easy to judge the failure mode and location as well as the failure limit for complex structural systems such as the piping system and for an active equipment and electrical equipment such as a pump.

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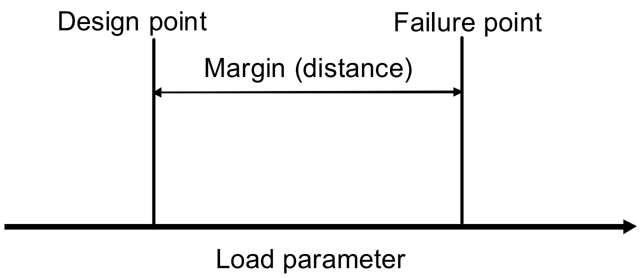

Fig. 1 Definition of the safety margin (distance between the design point and failure point)

Therefore, we need to collect information on the seismically induced failure of the equipment and systems during the strong earthquake ground motion. Based on the information, seismic performance is to be understood and the safety margins are quantified.

One of those activities are a series of seismic capacity tests[3] performed by Japan Nuclear Energy Safety Organization (JNES) in which the vibration table was used at higher acceleration than the design basis level. The risk-dominant acceleration level is usually far beyond the design basis level. The information, i.e. the seismic capacity tests and the seismic experience, would be valuable and effective for the seismic fragility update if it is utilized appropriately.

The cost of the seismic capacity tests is expensive. Thus when we update the seismic fragility, the seismic experience should be selected, and an additional qualification test should be designed so that the fragility update using the information is useful and cost-effective. Therefore, we need a mathematical methodology for the seismic fragility update and a metric for evaluating the importance of obtaining additional information.

The authors proposed a Bayesian approach for the fragility update[4] that can reduce the uncertainty in the seismic capacity evaluation. The Bayesian method and the mathematical process in the fragility parameter estimate are practical and transparent. The acceleration level of a seismic qualification test and the number of components tested are optimized by information entropy to reduce the uncertainty cost-effectively.[5] By comparing the entropy of two test programs, we can decide the better one. If we compare the expected entropy with respect to the seismic hazard uncertainty, it is shown that the importance or usefulness of the test for the epistemic uncertainty reduction is different for high seismic zone and low seismic zone. A pessimistic analyst and an optimistic analyst interpret the same test information in a different way.

However, the information entropy itself does not seem to be an absolute measure that explains the reduction of the uncertainty. In other words, we cannot tell how much extent the uncertainty can be reduced by obtaining new information. The uncertainty and the entropy are not quantitatively interrelated. The present study addresses the proposal of an absolute metric that connect the uncertainty reduction and entropy. In section 2, the seismic fragility model is described and features of the uncertainty in the fragility are discussed. Bayesian update process is described in section 3. The relationship of the information entropy and the uncertainty reduction is presented in section 4.

2. UNCERTAINTY OF SEISMIC FRAGILITY

2.1. Seismic Fragility and Uncertainties of Two Types

Whole the spectrum of seismic-induced scenario is taken into consideration in the seismic PSA of an NPP. Therefore, seismic fragilities need to be prepared for every failure mode of all the safety-related equipment and structures considered in the seismic PSA in order to quantify the residual seismic risk. Generally, a large number of systems and equipment are involved in the seismic PSA model. A problem is that our knowledge and data necessary for the fragility evaluation are not always complete and it is a difficult task to prepare the sufficient seismic fragility dataset specific to the NPP under consideration. If the seismic behavior of equipment is not well-understood or the uncertainty is large, additional seismic qualification tests would be performed to update the seismic fragilities and to reduce the uncertainty.

The seismic fragility is a concept that describes the vulnerability of equipment and a system in terms of a probability as a function of the earthquake intensity such as peak ground acceleration (PGA). It may be reasonable to use other seismic load parameters instead of the PGA to express the seismic vulnerability. A bending moment, a displacement and energy are examples of the parameter. It is noted, however, we need to introduce a model to connect the seismic intensity and the vulnerability parameter. In most cases, the PGA is used as the seismic intensity to express the seismic fragility.

The seismic fragility is defined as the conditional failure probability of the equipment or system as a function of the PGA as shown in Fig. 1. The fragility (conditional failure probability) increases as the PGA. Letting us define α as the PGA and C as the seismic capacity of the equipment, the conditional failure probability is expressed as the probability that α exceeds C.

$$F(\alpha) = \Phi\left[\frac{\ell n(\alpha/C)}{\beta_R}\right] \tag{1}$$

where $\Phi[\cdot]$ is the cumulative normal distribution function. As β_R is smaller, the fragility curve becomes steeper as shown by the dotted line in Fig. 1. The curve is gently-sloping if β_R is large, and it happens that the equipment does not fail in a large earthquake or does fail in a small earthquake and the results are unpredictable because of random characteristics. In other words, the threshold of the failure is not clear and the PGA may not be a suitable parameter of the seismic-induced failure characteristics. In this situation, we see that the seismic failure occurs randomly at a probability. We take the first letter from "randomness" to express β_R and call an aleatory uncertainty. To reduce β_R, we need to find an appropriate physical parameter that can explain the failure phenomenon.

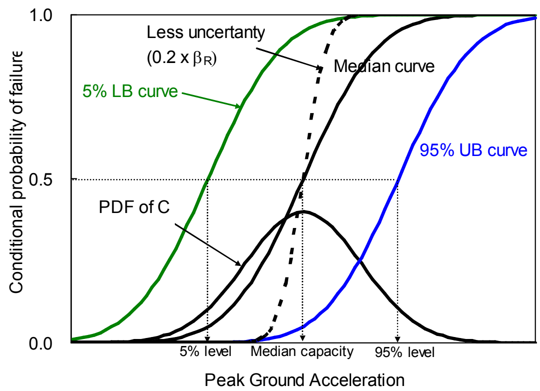

Fig. 2 Seismic fragility model and the PDF of the seismic capacity.

The median value of the seismic capacity C is called the median capacity A_m. We do not know the exact value of the seismic capacity until the equipment fails. Thus we assume a lognormal distribution for the random variable C.

$$f(C) = \frac{1}{\sqrt{2\pi}\beta_U C}\exp\left[-\frac{1}{2}\left\{\frac{\ell n(C/A_m)}{\beta_U}\right\}^2\right] \equiv \phi\left[\frac{\ell n(C/A_m)}{\beta_U}\right] \tag{2}$$

where $\phi[\cdot]$ is the normal probability density function (PDF). In Fig. 2, the PDF of C is also shown. Corresponding to the upper 95% bound and lower 5% bound of C, upper bound (UB) and lower bound (LB) fragility curves are illustrated. The variability of C reflect the fact that we do not know the seismic capacity because of the lack of knowledge about the physical characteristics of the capacity. If we can model the equipment and define the failure criterion exactly with perfect knowledge, we could define the failure point, i.e., seismic capacity, exactly.

Here, we can rewrite Eq. (1) as:

$$F(\alpha) = \Phi\left[\frac{\ell n(\alpha / A_m) - \ell n(C / A_m))}{\beta_R}\right] = \Phi\left[\frac{\ell n(\alpha / A_m) - \beta_U \cdot \Phi^{-1}(p))}{\beta_R}\right] \tag{3}$$

We use the relationship, $\ell n(C / A_m) = \beta_U \cdot \Phi^{-1}(p)$ from Eq. (2) where p is the confidence level concerning the seismic capacity. If $p = 0.05$, we can define 5% LB curve and $p = 0.95$ corresponds to the 95% UB curve in Fig. 2.

2.2. Seismic Fragility and Response-Strength Model

The seismic fragility concept is explained from the comparison of the response and the capacity. The structural integrity is quantified by comparing the stress and strength and is often called the stress-strength model. The two ideas are interchangeable each other. The stress-strength concept is based on the idea that the equipment fails when the stress caused by the seismic response of the equipment exceeds the strength. As shown in Fig. 2, the seismic stress and the strength are expressed as random variables. In the stress-strength model, the parameter may not be a PGA in gravity unit but ones that express the vulnerability of equipment appropriately. For example, it can be in Pa such as the applied stress by an earthquake and the ultimate strength. The seismic hazard concept is based on the earthquake parameter while the stress-strength concept relies on the capacity parameter. In the seismic PSA, the hazard curve is commonly expressed by the PGA.

Assuming that the response R and the strength S follow the lognormal distribution with the median value R_m and S_m, and logarithmic standard deviation (LSD) β_{Res} and β_{Str}, respectively, R/S is also lognormal. The median and the LSD are expressed as $\xi_m = R_m / S_m$ and $\beta_\xi = \sqrt{\beta_{Res}^{\ 2} + \beta_{Str}^{\ 2}}$, respectively. Therefore, the PDF of ξ is:

$$f(\xi) = \phi\left[\frac{\ell n(\xi / \xi_m)}{\beta_\xi}\right] \tag{4}$$

The failure criterion is $\xi \geq 1$ and the conditional failure probability is

$$F(\alpha) = \Phi\left[\frac{\ell n(\xi)}{\beta_\xi}\right] = \Phi\left[\frac{\ell n(R / S)}{\beta_\xi}\right] = \Phi\left[\frac{\ell n(R\alpha A_m / S\alpha A_m)}{\beta_\xi}\right] = \Phi\left[\frac{\ell n(\alpha / A_m) - \ell n(\alpha S / A_m R)}{\beta_\xi}\right] \tag{5}$$

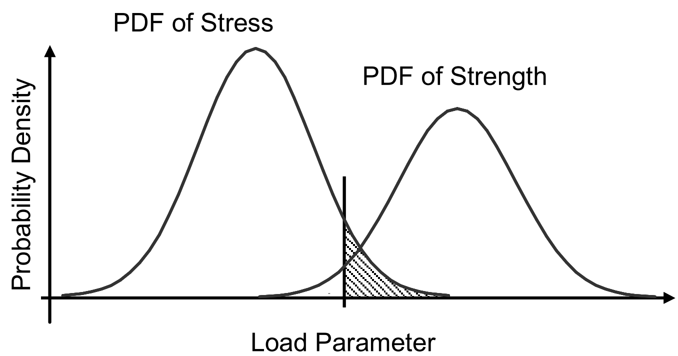

Figure 3 Stress-strength model for evaluating the failure probability

Comparing Eqs. (3) and (5), we find that:

$$\beta_U \Phi^{-1}(p) = \ell n \frac{S / R}{A_m / \alpha} = \ell n \frac{C}{A_m} \tag{6}$$

$$\beta_R = \beta_\xi = \sqrt{\beta_{Res}^{\ 2} + \beta_{Str}^{\ 2}} \tag{7}$$

It is understood that β_U reflects the modeling uncertainty of predicting the strength and the response in relation to the PGA. If the seismic model is perfect and we can predict the ratio of the seismic response to the strength, β_U disappears. The randomness β_R explains the composite variability of the response and the strength. The median curve explains all the randomness.

It is noted that the uncertainties considered in the seismic fragility model is equivalent to the response-strength model. The meaning of β_R and β_U is now clearly understood. The randomness β_R is the composite variability of the response and the strength (see Fig. 3). We can say that the seismic failure phenomenon cannot be explained by a single load parameter. The epistemic uncertainty β_U reflects the difficulty or lack of knowledge in predicting the response and strength from the earthquake motion. It involves the uncertainty in the model, theory, analytical technique, assumptions and data. In other words, the lack of knowledge can be improved as we obtain more information and better models developed.

2.3. Aleatory And Epistemic Uncertainties

The AESJ standard for the seismic PSA defines two uncertainties: aleatory uncertainty β_R and epistemic uncertainty β_U . The former is the variability of a quantity controlled by chance or randomness. The latter reflects the incompleteness of knowledge and information. Accordingly the epistemic uncertainty can be reduced by a new idea and data (information). It implies new information is directly related to the reduction of epistemic uncertainty.

The uncertainty is discussed in relation to the system modeling in USNRC Regulatory Guide 1.174[6] as such “there are two facets to uncertainty that, because of their natures, must be treated differently when creating models of complex systems.” It is common to term aleatory uncertainty and epistemic uncertainty.” According to RG. 1.174, “The aleatory uncertainty is that addressed when the events or phenomena being modeled are characterized as occurring in a "random" or "stochastic" manner, and probabilistic models are adopted to describe their occurrences. It is this aspect of uncertainty that gives PRA the probabilistic part of its name. The epistemic uncertainty is that associated with the analyst's confidence in the predictions of the PRA model itself, and it reflects the analyst's assessment of how well the PRA model represents the actual system being modeled. This has been referred to as state-of-knowledge uncertainty.” Further, the epistemic uncertainty is classified into three categories: parameter uncertainty, model uncertainty and completeness uncertainty.

According to the OECD[7], both of the aleatory uncertainty and epistemic uncertainty are expressed using the probability. The aleatory uncertainty results from the effect of “inherent randomness” or “stochastic variability”. The probability means the relative frequency in a number of independent random trials. The aleatory uncertainty is associated with the question “what can occur and with which probability”. The epistemic uncertainty results from “imperfect knowledge” regarding values of parameters. The probability distributions associated with uncertain parameters represents “state of knowledge” about the right values of the parameters. The probability is a degree of belief or confidence that a statement is true. Such probability distributions for uncertain parameters are often derived from expert judgment. The epistemic uncertainty can be associated with thw question “which value is the right one and how well do we know that.

Another discussion is given in the International Standard Organization for Standardization Guide to the Expression of Uncertainty in Measurement.[8] The ISO Guide describes that “the uncertainty evaluated from statistical analysis of a series of observation and based on frequency distributions is referred to as Type A. The uncertainty determined by judgment and based on a priori distributions is referred to as Type B. In both case the distributions are models that are used to represent the state of knowledge.” The reason that the two type is considered is “The purpose of the Type A and Type B classification is to indicate the two different ways of evaluating uncertainty components and is for convenience of discussion only. The classification is not meant to indicate that there is any difference in the nature of the components resulting from the two types of evaluation. Both types of evaluation are based on probability distributions, and the uncertainty components resulting from either type are quantified by variances or standard deviations."

The uncertainty definition in the SSE, RG1.174 and ISO Guide are analogous. All of them consider the uncertainties reflect our state-of-knowledge. The SSE and RG 1.174 decompose the uncertainty into lack of knowledge and the random error. The SSE termed the uncertainty related to our state-of-knowledge as bias. The RG1.174 calls epistemic uncertainty that is further classified into parameter uncertainty, model uncertainty and completeness uncertainty. The randomness is designated as the random error in SSE and aleatory uncertainty in RG1.174. The uncertainties are not further classified in the ISO Guide that deals with the measurement uncertainty. Although ease of mathematical handling is emphasized, the uncertainty originated from the state-of-knowledge. It seems the basic idea and concept are common and the uncertainty is expressed by probability.

3. BAYESISAN UPDATE OF SEISMIC FRAGILITY

One can update the seismic fragility by performing shaking table tests, structural response analyses with advanced method or utilizing seismic experience. It is noted that the shaking table test is costly. The detailed analysis using high performance computer is also expensive as well. Thus we have to decide the most cost-effective test or analytical conditions based on the engineering judgment.

Our knowledge on the seismic-induced failure is imperfect and its uncertainty is expressed by a PDF as in Eq. (2). When we have an empirical evidence E, it is reasonable that we update the function based on the new information. Let $L(E|C)$ be the likelihood of the evidence E on condition that we have a prior PDF $f(C)$ for the seismic capacity C, the posterior PDF of the seismic capacity after we obtain the evidence E is expressed as:

$$f(C|E) = \frac{f(C)L(E|C)}{\int_0^\infty f(C)L(E|C)dC} \tag{8}$$

This is the Bayes theorem and the process is called "Bayesian update".

Let us consider a situation where a shaking table test is performed at acceleration level α. If an analyst believes the seismic capacity of the component is C, and the randomness is β_R, the failure probability is given by Eq. (1) on condition that the seismic capacity is C, i.e. $F(\alpha|C)$. The seismic capacity is an uncertain variable with the median capacity A_m and the LSD β_U as seen from Eq. (2). When N components are tested at the same acceleration level α, there are $N+1$ possibilities. The likelihood L_k is the probability that k components fail out of N trials and is calculated by the binomial distribution:

$$L_k(\alpha, N|C) = \binom{N}{k} F(\alpha|C)^k \left\{1 - F(\alpha|C)\right\}^{N-k}. \tag{9}$$

Substituting Eqs. (2) and (9) into Eq. (8), one obtains a mathematical expression of the Bayesian update.

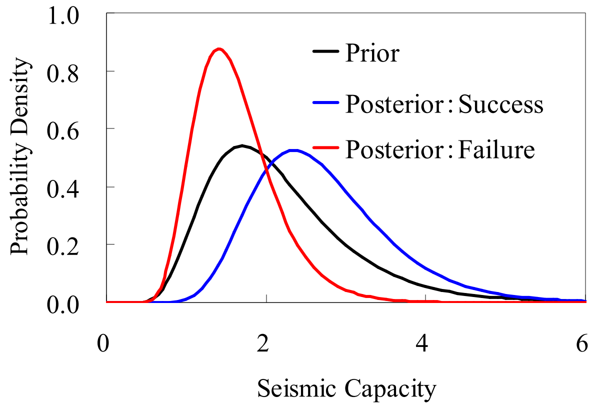

Fig. 4 Bayesian update: prior and posterior PDFs of the seismic capacity.

Let us consider a shaking table test performed at the median capacity level as an example. The prior fragility parameters are assumed to be $A_m = 2.0$, $\beta_R = 0.3$, and $\beta_U = 0.4$. In this case, $N = 1$ in Eq. (9) and we have two possibilities, i.e. success or failure. The likelihood function is either:

$$L_0(\alpha|C) = 1 - F(\alpha|C) \text{(success),or} \tag{10}$$

$$L_1(\alpha|C) = F(\alpha|C) \text{ (failure).} \tag{11}$$

Figure 4 shows the results of the Bayesian update of the seismic capacity. Two cases are considered in Fig. 4; the red line shows the case in which the component failed; the blue line corresponds to the successful test with no failure. It is seen that the PDF of the seismic capacity is substantially changed according to the information obtained from a single component test at the median capacity PGA level.

4. INFORMATON ENTROPY AND EPISTEMIC UNCERTAINTY REDUCTION

Here a question arises about the fragility qualification by a shaking table test. What acceleration level should be selected in the shaking table test? How many components should be tested? The logarithmic likelihood and the information entropy will give the answer to the questions. It is interesting to know the importance or worth of new information from the viewpoint of the fragility updating. The importance of a new evidence can be evaluated by the logarithmic likelihood V_k:

$$V_k(\alpha, N|C) = -\ell n L_k(\alpha|C). \tag{12}$$

where L_k is the likelihood. The likelihood L_k is the probability that the evidence is observed and is given by Eq. (4) in the present case.

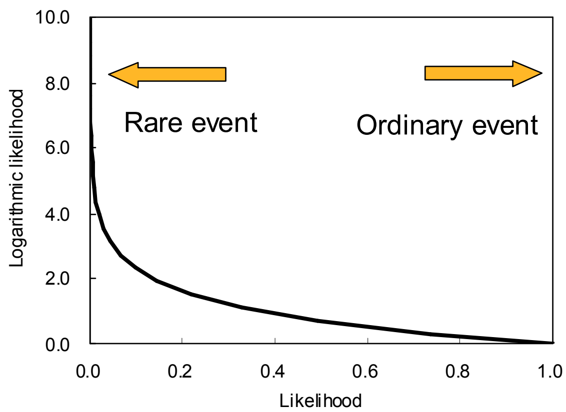

Fig. 5 Bayesian update: prior and posterior PDFs of the seismic capacity.

As in Fig. 5, the logarithmic likelihood varies from zero to infinity. If the component is very rigid and seismic-resistant, the likelihood of component failure is very small and the logarithmic likelihood approaches infinite. It implies that the failure of a very resistant component is a rare event. Viewing it from the other side, we understand that an occurrence of a rare event is very valuable and the importance of the information would be significant. On the other hand, the logarithmic likelihood is zero if the failure probability or the likelihood is unity. We take it granted for that a very fragile component should fail in the shaking table test and no noteworthy information is obtained from the test. Therefore, the logarithmic likelihood is zero which implies the test result is almost meaningless or the information value is negligibly small. It can be said that the logarithmic likelihood reflects the information quantity and is a useful measure to judge the test result is notable or not after the test.

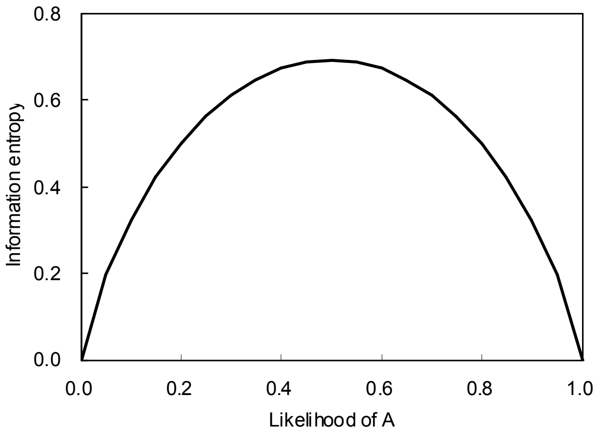

Fig. 6 Information Entropy of a binary trial.

Since we do not know the test results in advance, the expected value of the logarithmic likelihood with respect to all the possibilities is a point of concern. It is defined as the information entropy, E. We assume the test result is either fail or success. The conditional information entropy is defined as:

$$E\left(\alpha|C\right) = -F\left(\alpha|C\right)\ell nF\left(\alpha|C\right) - \left\{1 - F\left(\alpha|C\right)\right\}\ell n\left\{1 - F\left(\alpha|C\right)\right\}, \tag{13}$$

on condition that the capacity is given. When N components are tested at the same acceleration level, there are $N+1$ possibilities. Thus the general expression of the conditional entropy is:

$$E\left(\alpha, N|C\right) = -\sum_{k=0}^{N} L_k \ell n L_k \ . \tag{14}$$

The importance of the additional information on the seismic fragility update is influenced by the subjective judgment on the seismic capacity of the analyst since the logarithmic likelihood and the information entropy depends on the seismic capacity. Thus the importance is quantified by taking average of the logarithmic likelihood and the information entropy in terms of the seismic capacity PDF.

It is reasonable that the expected information entropy with respect to the seismic capacity should be utilized to evaluate the importance of the additional information such as a vibration test or seismic experience. The expected values of the logarithmic likelihood $\bar{V}_k(\alpha)$ and the information entropy $\bar{E}(\alpha)$ are given by:

$$\bar{V}_k(\alpha) = \int_0^\infty V_k\left(\alpha|C\right)f(C)dC = -\int_0^\infty f(C)\ell nL_k\left(\alpha|C\right)dC \text{, and} \tag{15}$$

$$\bar{E}(\alpha) = \int_0^\infty E\left(\alpha|C\right)f(C)dC = -\int_0^\infty f(C)\sum_{k=0}^{N} L_k\left(\alpha|C\right)\ell nL_k\left(\beta|C\right)dC \ . \tag{16}$$

The expected entropy is interpreted as the importance of a test at acceleration level α before the test is performed for the analyst who believes the seismic capacity PDF is $f(C)$. On the other hand, the expected logarithmic likelihood is the importance of the test result that k components failed at acceleration level α for the analyst who believes the seismic capacity PDF is $f(C)$.

The expected entropy tells us how the fragility estimate is efficiently updated with information or evidence. In other words, additional test for the seismic fragility update should be designed so that the entropy becomes the maximum. Figure 6 shows the information entropy as a function of a binary trial A. By taking the average with respect to the likelihood, the entropy becomes the maximum when the likelihood of the binary trial is even (i.e. the occurrence probability is 0.5).

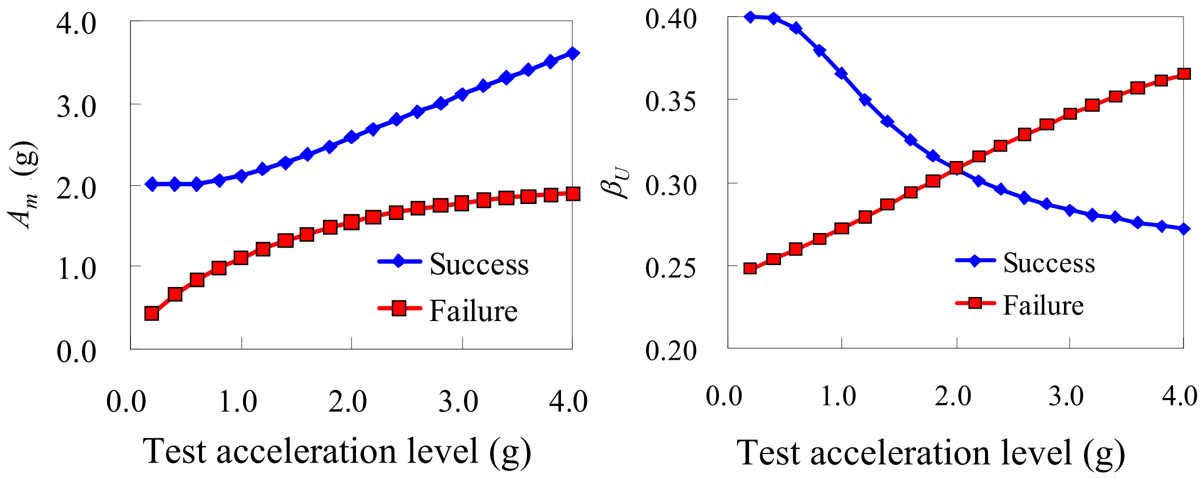

Fig. 7 Bayesian update of fragility parameters by a binary trial. The test result is either success or failure. The posterior median capacity A_m (left figure) and logarithmic standard deviation β_U (right figure) are shown as a function of test acceleration level.

The PDF of the seismic capacity is updated according to Eq. (4) and is shown in Fig. 3. Figure 6 illustrates how the fragility parameters, i.e. the median capacity A_m (in the left figure) and the logarithmic standard deviation β_U (in the right figure) are improved by the Bayesian updating process by obtaining additional information from a shaking table test at the test acceleration level α. The prior parameters are $A_m = 2.0g$ and $\beta_U = 0.4$, respectively. In the binary trial, the consequence is either success or failure. The red lines are posterior parameters for failure and the blue lies are for success. It is noted that β_U definitely decreases once one has more information. As discussed earlier, β_U reflect the epistemic uncertainty on the fragility, i.e. lack of knowledge concerning the seismic performance of a component. The reduction of β_U depends on the test acceleration level and the test results. What we see is that unexpected results (failure in low acceleration or survival in high acceleration) present greater reduction of the uncertainty. Since the component failure is very unlikely at a low acceleration level or survival at a high acceleration level is rare, the median capacity A_m is improved significantly in the extreme case. However in average with respect to the likelihood, A_m changes slightly,

Here let us discuss the epistemic uncertainty of the fragility and take average of β_U regarding the likelihood. Figure 7 shows the prior LSD (red dotted line) and posterior LSDs for success (violet dashed line) and failure (green dashed line). Of course the prior LSD is constant irrespective to the test level. The blue line is the expected posterior LSD that is the average of two posterior LSDs. It is noted that the average LSD approximate to the LSD after success in low test acceleration level because survival is very plausible. On the other hand, we see that the average LSD gets close to thr LSD after failure. The expected LSD is the minimum at $\alpha = A_m$ when the test acceleration level equals the median capacity and the results is most uncertain. Also shown in Fig. 7 is the expected entropy calculated by Eq. (12). It implies the utility of a shaking table test at the test acceleration level when the seismic capacity is estimated to be $f(C)$. It is emphasized that the utility is maximum when the test acceleration level is $\alpha = A_m$ which is coincident with the acceleration level where the reduction of the epistemic uncertainty is largest. Now we understand that the test should be performed at the load level that gives the maximum expected entropy. The expected entropy may be called a the subjective entropy because it is the entropy profile for a person who believe the seisimic capcity is expressed as $f(C)$.

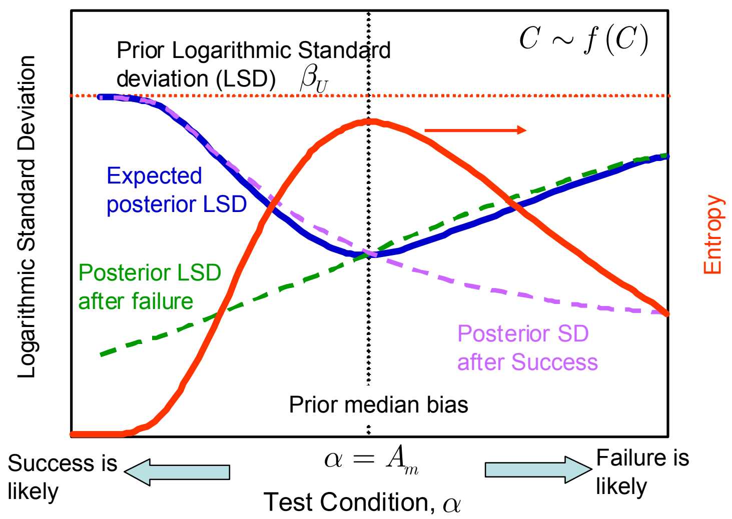

Fig. 8 Bayesian update: prior and posterior PDFs of the seismic capacity.

Based on the expected or subjective entropy we can decide the best acceleration level for the shaking test. The expected entropy is a quantitative measure of the epistemic uncertainty reduction. Next the authors discuss the relationship of the entropy and the epistemic uncertainty. The total entropy for an even event with the probability 0.5 is obtained from Eq. (9) as:

$$E\left(A_m,1\middle|C\right)=-\sum_{k=0}^{1}L_k\ln L_k=0.693 \tag{17}$$

Let us consider a test of two components which median capacity is $A_m = 2.0g$ in common and the LSD is $\beta_U = 0.4$ and $\beta_U = 0.1$, respectively. The expected entropy and the expected reduction of the epistemic uncertainty are shown in Fig. 9. When we perform a test at the median capacity level, the entropy is approximately 0.4 for the component of $\beta_U = 0.43$, i.e. 62 percent of the maximum entropy given in Eq. (17). On the other hand, it is 0.66 for the component of $\beta_U = 0.1$, which is 95 percent of the maximum entropy. It is seen from Fig, 9 that the relative magnitude of the expected entropy and the uncertainty reduction is different by the prior uncertainty. If the prior uncertainty is large ($\beta_U = 0.4$), the uncertainty reduction is larger in comparison to the expected entropy.

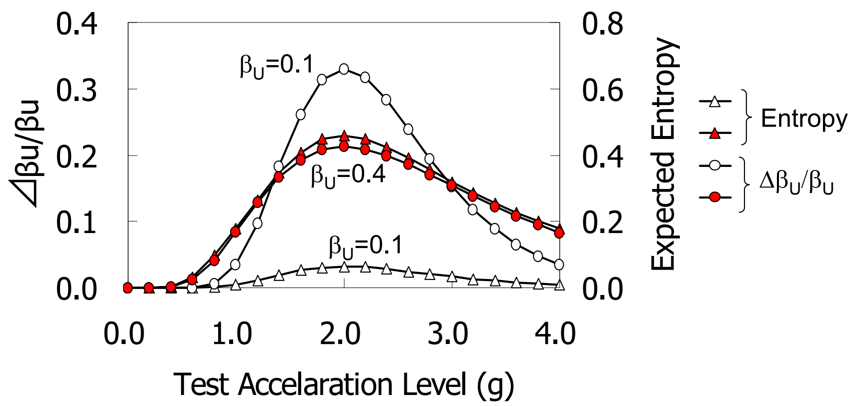

Fig. 9 Uncertainty reduction effect and the expected entropy of thought test.

If the LSD is not large, it can be shown that the expected entropy is related to a reduced uncertainty λ defined by the following equation:

$$\lambda = \frac{1}{{\beta_U}^2}, \qquad \Delta\lambda \cong \frac{2\Delta\beta_U}{{\beta_U}^3} \tag{18}$$

Figure 10 is a plot of the expected entropy and $\Delta\lambda$ for the same components as in Fig. 9. From this figure, we can see the relationship of the uncertainty and the expected entropy is consistent for components with different β_U . Here we can relate the uncertainty reduction and the entropy. A shaking table test can be designed in accordance with uncertainty reduction that would be achieved by the additional data.

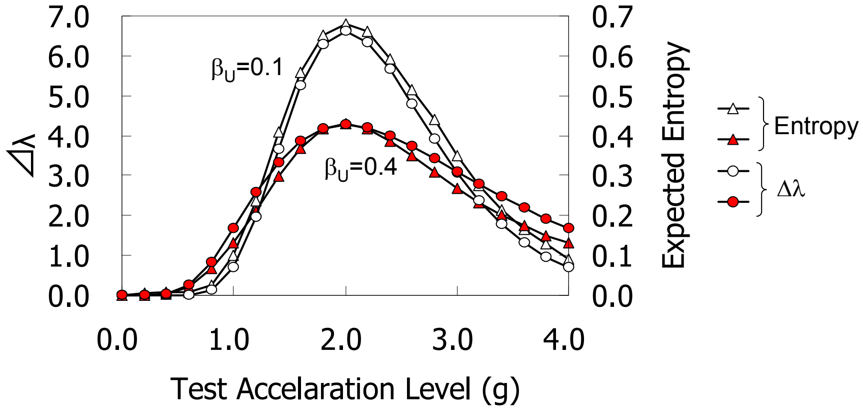

Fig. 10 Reduced uncertainty reduction effect and the expected entropy of thought test.

5. CONCLUSION

It is a great concern how effectively the epistemic uncertainty can be reduced in the PSA as the safety research advances and new knowledge is obtained. The only way to reduce the epistemic uncertainty seems to be to extend our knowledge concerning the uncertain phenomena or parameters. If we have new experimental data, the evidence may suggest improvement or modification in the model and the database for the safety analysis. The issue is how to design the new experiment so that we can effectively improve the model and the database.

It is known that the importance of the information is measured with the information entropy. In the present study, the authors predict the epistemic uncertainty reduction on the basis of the information entropy concept. The prior and the posterior seismic fragilities are connected using the Bayesian method. The present approach is applied to the seismic PSA of nuclear power plant in which a limited number of components are estimated to be risk contributors. It is reasonable and effective to collect the seismic fragility data concerning the contributors. On the other hand, if the importance of the information is small, we take little notice of the data. We can determine the experimental conditions that enhance our knowledge on the phenomena in the most effective and economical way. As a result, the uncertainty of the seismic PSA results is reduced on the basis of metrics expressed as the information entropy.

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