Stochastic Analysis of Natural Circulation Decay Heat Removal of Sodium Cooled Fast Reactor based on Latin Hypercube Sampling

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Abstract: A stochastic analysis of uncertainty correlation between input variables and analytical result of a computational simulation has been performed in the present study. A natural circulation decay heat removal in a sodium cooled fast reactor is chosen as a target phenomenon. The Midpoint Latin Hypercube Sampling (MLHS) and the correlation ratio are used to evaluate the rank of input variables upon the phenomenon. A common uncertain factor among the input variables has also been considered in the random sampling. As a result, it is concluded that the uncertainty correlation can be evaluated with the present method even when the common uncertain factor exists in some input parameters.

Keywords: Best Estimate Plus Uncertainty (BEPU), Latin Hypercube Sampling (LHS), Correlation Ratio, Sodium Cooled Fast Reactor

1. INTRODUCTION

In a safety assessment of Light Water Reactor (LWR), a stochastic analysis method, such as Best Estimate Plus Uncertainty (BEPU), has been coming to be widely used. For instance, the United State Nuclear Regulatory (USNRC) has developed the Code Scaling, Applicability and Uncertainty method (CSAU) [1] that includes a systematic approach for the evaluation of uncertainty of safety related variables in a computational simulation.

In the BEPU method, an influence of each input variable on the analytical output has been investigated using a stochastic approach. Hence the number of total code runs will increase exponentially when the number of the reviewing input variables increases. Besides, it is commonly said in a computational simulation that the input variables are nonlinearly correlated each other in governing equations of the computation. In order to avoid the exponential increase of code run and to select the influential input variables adequately, it is of importance to evaluate the influential input variables on the computational result quantitatively with a small number of code runs. For example, the Phenomena Identification and Ranking Table (PIRT) process is introduced in the CSAU methodology.

Fast reactor has a great potential to prolong the total amount of nuclear fuel resource dramatically because it can generate plutonium from unburned uranium isotope (^{238}U). Consequently, the development of fast reactor technology will be a key focus for nuclear fuel cycle system. In the feasibility study of the commercialization of the fast reactor cycle system in Japan [2], a sodium cooled fast reactor is selected as the most promising reactor design. As a result of the probabilistic safety assessment of the sodium cooled fast reactor [3], it was concluded that a Loss of Heat Sink accident (LOHS) is one of the most risk-dominant sequence and hence a natural circulation Decay Heat Removal (DHR) plays an important role for the risk reduction.

Since a stochastic analysis method has not been well established in a safety assessment of sodium cooled fast reactor, a stochastic analysis of natural circulation DHR has been carried out so as to investigate the influential input parameters on the phenomenon numerically in the present study. As a sampling manner and a quantification of the influence, the Midpoint Latin Hypercube Sampling (MLHS) and the correlation ratio, originally proposed by McKay [4], are introduced respectively.

Furthermore, the uncertainty of input variable is segmented into two parts; one is an independent uncertainty that corresponds to a random factor of each input variable. The other is a common uncertain factor among the specific input variables such as a heat transfer coefficient and pressure loss coefficient. The common uncertain factor comes from a common background that is refereed when the value of the input parameter is determined.

2. STOCHASTIC ANALYSIS METHOD

2.1 Correlation Ratio

According to McKay [4], the variance of code output (y) can be divided into two portions in the following.

$$V[y] = V[E(y \mid \mathbf{x}_S)] + E(V[y \mid \mathbf{x}_S]) . \quad (1)$$

Here, V[·] and E[·] indicate the variance and the expected value respectively. **x** means the input variable vector and hence $V[y|\mathbf{x}_S]$ and $E(y|\mathbf{x}_S)$ denote the variance and expectation on condition that a subset $\mathbf{x}_S$ of the input vector is fixed.

The first term of the right hand side of Eq. (1) is named as Variance of Conditional Expectation (VCE). The VCE means the magnitude of the correlation of the input parameter $\mathbf{x}_S$ and y and is defined as:

$$V[E(y \mid \mathbf{x}_S)] = \int (E(y \mid \mathbf{x}_S) - E(y))^2 f_{\mathbf{x}_S}(\mathbf{x}_S) d\mathbf{x}_S . \quad (2)$$

Where $f(\cdot)$ denotes the Probability Density Function (PDF) of the variable.

The second term in the right hand of Eq. (1) means the within-group variance or the residual defined as:

$$E(V[y \mid \mathbf{x}_S]) = \iint (y - E(y \mid \mathbf{x}_S))^2 f_{y|\mathbf{x}_S}(y) f_{\mathbf{x}_S}(\mathbf{x}_S) dy d\mathbf{x}_S . \quad (3)$$

The residual reflects the variation of the individual data within the group from the group-mean value and thus it has nothing to do with $\mathbf{x}_S$. Accordingly, one can consider that Eq. (3) reveals the residual with respect to the uncertainty of $\mathbf{x}_S$.

Accordingly, the correlation ratio ($\eta_{\mathbf{x}_S}$) that indicates the relative importance of the input parameter uncertainty with respect to the output uncertainty can be defined as a proportion of the VCE to the output variance V[y] as shown in the following.

$$\eta_{\mathbf{x}_S} = \frac{V[E(y \mid \mathbf{x}_S)]}{V[y]} . \quad (4)$$

Figure 1 shows a schematic of the VCE and the residual in the code output. The color of the line corresponds to the level of the focused input parameter ($\mathbf{x}_s$). The bold line of each color means the expected value of each level ($E(y|\mathbf{x}_S)$). Hence the variance of the bold lines reveals the VCE. On the other hands, solid circles in Fig. 1 indicate the variance which has nothing to do with $\mathbf{x}_S$ because of the constant value of $\mathbf{x}_S$ in the same colored lines. Then the expected value of each circle ($V[E(y|\mathbf{x}_S)]$) represents the residual.

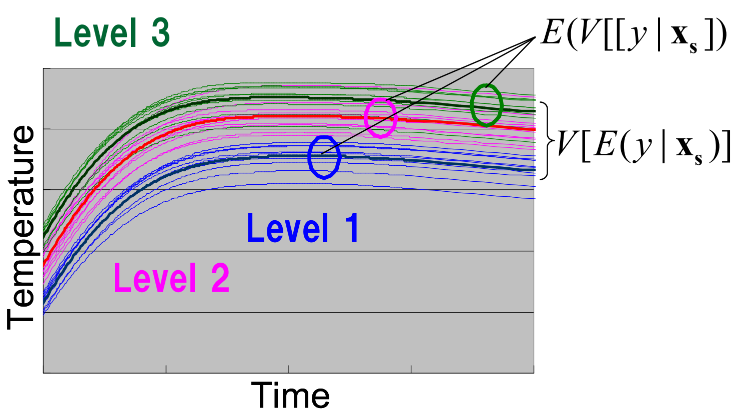


Fig.1: Schematic of VCE and residual

2.2 Midpoint Latin Hypercube Sampling

The schematic of LHS is depicted in Fig. 2. For simplicity, only two input parameters are shown in Fig. 2. In the LHS, the Probabilistic Density Function (PDF) of each input parameter ($\mathbf{x_S}$) is divided into n strata of equal marginal probability (=1/n, n = 4 in Fig. 2). Next, a random sampling is done once from each stratum. As seen in Fig. 2, an input variable set of the same level never appears in the LHS as a design matrix. In the normal LHS, additional random samplings are carried out to define each sampling point within the section of the equal marginal probability. On the contrary, the center of the section is defined as the sampling point in the Midpoint LHS (MLHS) [5] as shown in the red lines in Fig.2 for simplicity.

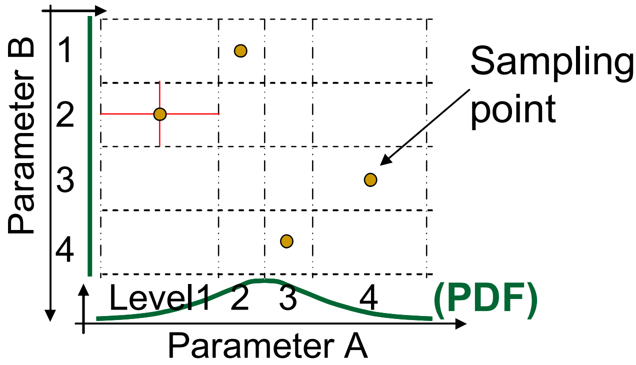


Fig.2: Schematic of LHS

The design matrix consists of $n \times s$. Here s is the number of input variables. Then the replicate of the matrix is generated based on the design matrix by applying the Latin square design where the cumulative stratum between the input variables does not appear in a same code run.

2.3 Common Uncertain Factor

In case of some input parameters such as a heat transfer coefficient and a pressure loss coefficient, an uncertainty random factor can be divided into an independent uncertainty and a common uncertainty. The former comes from independent characteristics of each component such as a size, location, material and manufacturing process. The common uncertainty will come from a common background, such as an empirical correlation and working fluid, which is referred when the value is determined.

Let us consider the input variable of component K (x_K). For simplicity, it is assumed that x_K follows the normal distribution with the mean μ_K and the standard deviation σ_K as:

$$x_K \sim N(\mu_K, \sigma_K)\,. \tag{5}$$

When one takes into account the independent factor and the common factor, the input variable can be written as the summation of each factor as shown in the following.

$$x_K = x_K^I + x_K^C\,. \tag{6}$$

Here superscripts I and O mean the independent and the common respectively. Considering the probabilistic features of the dependency, Eq. (5) will be rewritten as:

$$\begin{aligned} x_K &\sim N(\mu_K, \sigma_K) = N\left(\mu_K, \sqrt{\eta_K^{I^2} + \eta_K^{C^2}}\right), \\ \sigma_K^2 &= \eta_K^{I^2} + \eta_K^{C^2}\,. \end{aligned} \tag{7}$$

Where, η is the variance of each factor. Then we introduce the fraction of the common uncertainty (ϕ) as:

$$\eta_K^{I^2} = (1-\phi)\sigma_K^2,\; \eta_K^{C^2} = \phi\sigma_K^2\,. \tag{8}$$

It is easy to generate samples of x_K following Eq. (7). Standard normal random sample r_K and $\hat{r}$ that are independent each other are generated. Then, the sample of x_K is obtained by:

$$\tilde{x}_K = \mu_K + \eta_K^I r_K + \eta_K^C \hat{r} . \tag{9}$$

Since the mean values of r_K and $\hat{r}$ and the standard deviations are set to be zero and unity respectively, the mean and the variance of Eq. (9) becomes μ_K and σ_K^2 respectively. Consequently, Eq. (9) coincides with Eq. (5). It is also noted that the present procedure of the common uncertain factor is easy to be extended to more components by applying the $\hat{r}$ in the sampling.

2.4 Discretization Method

When one focuses on the input variable x_i, the sample average of the output (y) for $x_i = x_{ij}$ is expressed as:

$$\overline{y}_j = \frac{1}{r}\sum_{k=1}^{r} y_{jk} . \tag{10}$$

Here r is the number of the replicate. Then the expected value of the variance of $\overline{y}_j$ is calculated as:

$$\mathrm{E}(\mathrm{V}[\overline{y}_j \mid x_i]) = \frac{1}{n}\sum_{j=1}^{n} (\overline{y}_j - \overline{y})^2 \ , \tag{11}$$

where $\overline{y}$ means the average of all output. Equation (11) is rewritten in the following.

$$\begin{aligned}\mathrm{E}(\mathrm{V}[\overline{y}_j \mid x_i]) &\simeq \mathrm{V}[\mathrm{E}(\overline{y}_j \mid x_i)] + \mathrm{E}(\mathrm{V}[\overline{y}_j \mid x_i]) \\ &= \mathrm{V}[\mathrm{E}(y \mid x_i)] + \frac{1}{r}\mathrm{E}(\mathrm{V}[y \mid x_i]) = \mathrm{VCE}(x_i) + \frac{1}{r}\mathrm{E}(\mathrm{V}[y \mid x_i]).\end{aligned} \tag{12}$$

The expected value of the variance of y for which $x_i = x_{ij}$ is obtained as:

$$\mathrm{E}(\mathrm{V}[y \mid x_i]) = \frac{1}{nr}\sum_{j=1}^{n}\sum_{k=1}^{r} (y_{jk} - \overline{y}_j)^2 . \tag{13}$$

The VCE for the input variable x_i is then expressed by substituting Eqs. (11) and (13) into Eq. (12) as:

$$\mathrm{VCE}(x_i) = \frac{1}{n}\sum_{j=1}^{n} (\overline{y}_j - \overline{y})^2 - \frac{1}{nr^2}\sum_{j=1}^{n}\sum_{k=1}^{r} (y_{jk} - \overline{y}_j)^2 . \tag{14}$$

The variance of the output is also obtained in the following.

$$\mathrm{V}[y] = \frac{1}{nr}\sum_{j=1}^{n}\sum_{k=1}^{r} (y_{jk} - \overline{y})^2 . \tag{15}$$

Finally, the correlation ratio as shown in Eq. (4) is dicretized as:

$$\eta_{X_i} = \frac{\frac{1}{n}\sum_{j=1}^{n} (\overline{y}_j - \overline{y})^2 - \frac{1}{nr^2}\sum_{j=1}^{n}\sum_{k=1}^{r} (y_{jk} - \overline{y}_j)^2}{\frac{1}{nr}\sum_{j=1}^{n}\sum_{k=1}^{r} (y_{jk} - \overline{y})^2} . \tag{16}$$

It is noted that the VCE must be positive value theoretically as shown in Eq. (4). However, it might be negative in the discrete manner as in Eq. (16). The negative vale of the VCE will appears when the focused input variable has no influence on the output.

3. NUMERICA INVESTIGATION

3.1 Natural Circulation Decay Heat Removal in Sodium Cooled Fast Reactor

A typical system of loop type sodium cooled fast reactor is shown in Fig. 3. The system consists of the reactor core (vessel), the primary heat transport system and the secondary heat transport system. The intermediate heat exchanger (IHX) is installed to transport energy generated at the core from the primary system to the secondary system. The primary and secondary heat transport systems are filled with liquid sodium. In the secondary transport system, a steam generator is installed to activate a turbine and to generate electricity.

As shown in Fig. 3, an air cooler (Auxiliary Cooling System) is also installed in the secondary transport system in order to remove a decay heat. After a reaction protection system is activated, the coolant flow path is switched from the steam generator to the air cooler.

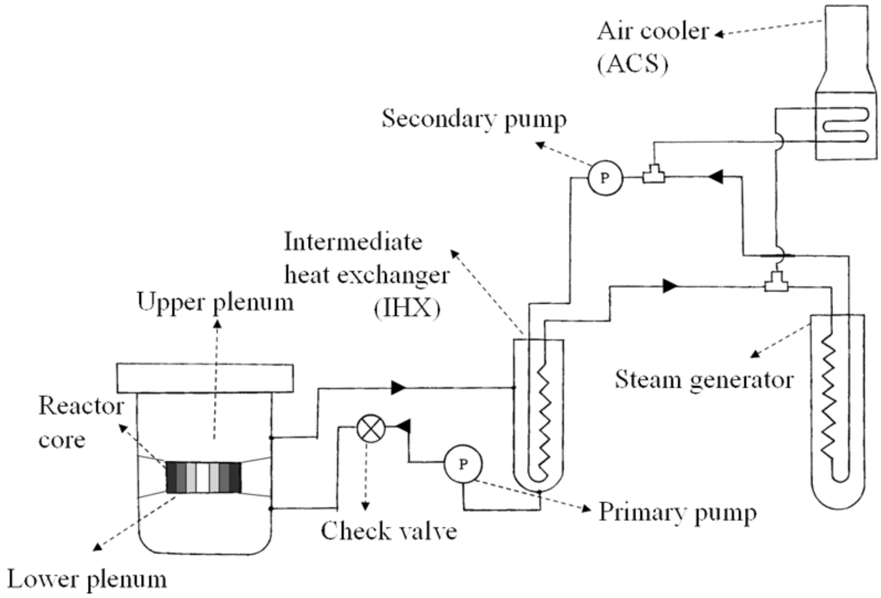
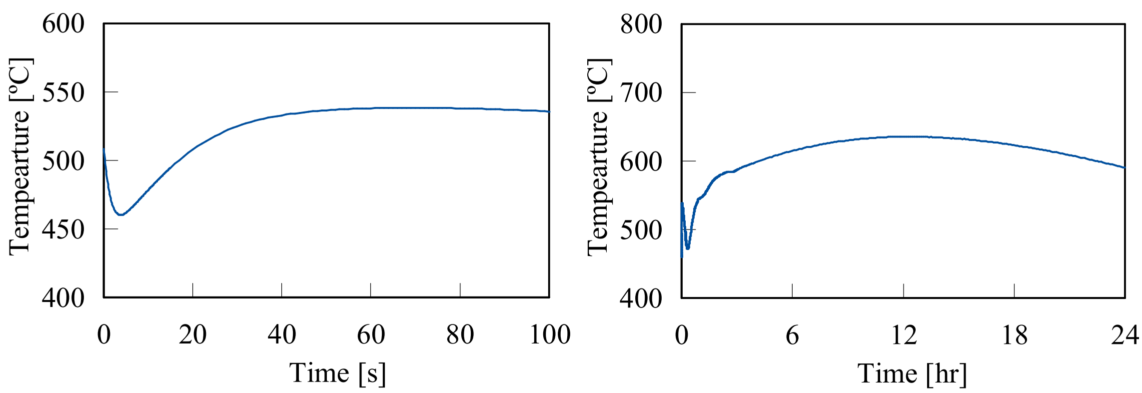


Fig. 3: Schematic of loop type sodium cooled fast reactor

Since the elevation of the IHX is higher than the reactor core and the ACS is higher than the secondary heat transport system, a natural circulation DHR can be achieved. The decay heat of the reactor core will be removed at the air cooler via the primary transport system, the IHX and the secondary transport system.

Figure 4 shows typical transients of the coolant temperature at a) core outlet in a short period of time and b) upper plenum in a long period time. In the analysis, the most appropriate input variables are used. When the reaction protection system is activated, several local maximum will appear in terms of the coolant temperature. Due to the mismatch of operation time between the power down of the core output and the pump stop, the primary local maximum of the coolant temperature appears just after the activation (less than one second). Then the secondary maximum will appear after several tens second because the natural circulation develops as shown in Fig. 4 a). On the other hands, the plant system is large and a huge amount of the coolant exists in the plant so that it takes several or more hours to achieve the total DHR through the whole system. Therefore the third maximum of the coolant temperature will appear after several hours as seen in Fig. 4 b). It is noted that the third maximum will not occur when the ACS has a sufficient capability of heat removal.



a) Short period (0-100s) at core outlet **b) Long period (0-24hr) at upper plenum**

Fig. 4: Typical transient of coolant temperature

In the present paper, we focus on the secondary and thirdly local maximum in the stochastic analysis. From the safety assessment viewpoint, the cladding temperature of the fuel and the structure temperature of the primary coolant boundary are of importance in the short and long periods respectively. Hence the coolant temperature of the core outlet is chosen as a code output in the short period as well as the coolant temperature of the upper plenum in the long period.

3.2 Numerical Investigation

In the stochastic analysis, a thermal-hydraulic plant response analysis code, LEDHER [6] is applied. As concerns the input variables, 18 parameters are considered in the present study. The parameters and their statistical properties are summarized in Table 1.

Table 1: Input variables and statistical properties

Input variable	PDF	Uncertain factor (%)
Reactor core pressure loss	Log-normal	100
Core decay heat	Normal	10
Gap conductance of fuel	Normal	80
Heat transfer coefficient of fuel pin surface	Normal	50
Check valve pressure loss	Log-normal	100
Primary pump pressure loss	Log-normal	150
Primary flow coast down time constant	Normal	50
Secondary pump pressure loss	Log-normal	150
Secondary flow coast down time constant	Normal	50
Steam generator pressure loss	Log-normal	100
ACS pressure loss (coolant side)	Log-normal	150
ACS air flow rate	Normal	10
ACS air inlet temperature	Uniform	50
ACS heat transfer coefficient	Normal	50
IHX pressure loss (primary side)	Log-normal	150
IHX pressure loss (secondary side)	Log-normal	150
IHX heat transfer coefficient (primary side)	Normal	50
IHX heat transfer coefficient (secondary side)	Normal	50

In Table 1, the uncertain factor corresponds to $\pm 2\sigma$ in case of the normal distribution, whereas it indicates the variation scale in the logarithmic normal distribution. For instance, 100% of uncertain factor in the logarithmic distribution shows the range from $1/(1+1)\mu$ to $(1+1)\mu$ (here μ is the median).

As shown in Table 1, a large variation is considered in terms of pressure loss coefficients. This is attributed the fact that an empirical correlation of the pressure loss has been established mainly in a forced convection state. In a natural circulation of the large plant, a stable flow pattern will not be achieved resulting in the large variation. It is noted that the quantification of the variance in each input parameter is still challenging issue especially in the fast reactor safety assessment.

Table 2 shows the correlation of the common uncertain factor (ϕ in Eq. (8)) applied in the present study. The value of ϕ is set as +: $\phi = 0.2$, ++: $\phi = 0.5$ and +++ $\phi = 0.8$. The color of each symbol indicates the same group. Hence seven common factors are taken into account in the present study. Again it is mentioned that a mature discussion will be necessary for the determination of the common factor and its value as well as in the variation of each input variables.

Table 2 Correlation of common uncertain factor

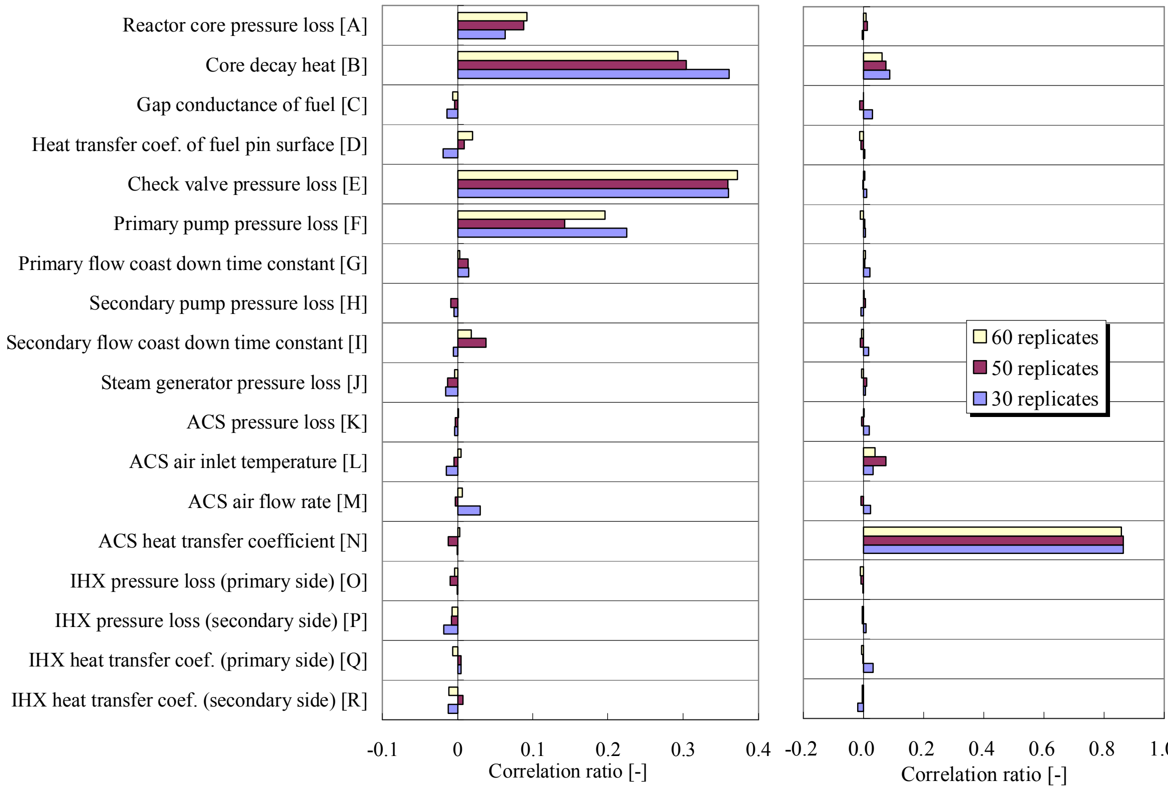
	Input parameter	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
A	Reactor core pressure loss								+		+	+				+	+		
B	Core decay heat																		
C	Gap conductance of fuel																		
D	Heat transfer coef. of fuel pin surface														+			+	+
E	Check valve pressure loss	+					+		+		+	+				+	+		
F	Primary pump pressure loss	+				+			+ +		+	+				+	+		
G	Primary flow coast down time constant									+ + +									
H	Secondary pump pressure loss	+				+	+ +				+	+				+	+		
I	Secondary flow coast down time constant							+ + +											
J	Steam generator pressure loss	+				+	+		+			+				+ +	+		
K	ACS pressure loss	+				+	+		+		+					+	+ +		
L	ACS air flow rate																		
M	ACS air inlet temperature																		
N	ACS heat transfer coefficient				+													+ +	+ +
O	IHX pressure loss (primary side)	+				+	+		+		+ +	+					+		
P	IHX pressure loss (secondary side)	+				+	+		+		+	+ +				+			
Q	IHX heat transfer coef. (primary side)				+										+ +				+ +
R	IHX heat transfer coef. (secondary side)				+										+ +			+ +	

In the analysis, the number of sample (n) in each input parameter is set to 10. Hence the design matrix consists of 10×18 (10 code runs in the matrix). The number of replicate (r) is chosen as a parameter and is set to 30, 50 and 60 which means the code runs ($n \times r$) of 300, 500 and 600 respectively.

Figure 5 shows the comparison of the correlation ratio where the common uncertain factors are not taken into account. In case of the short period (0-100s), the most influential input variable is the pressure loss coefficient of the check value that is installed at the primary heat transport system (see Fig. 3), followed by the core decay heat, the pressure loss coefficient of the primary pump and the reactor core. As seen in Fig. 5 a), the uncertainty of the pressure loss coefficient affects the output uncertainty dominantly. This is attributed the fact that the secondary local maximum of the coolant temperature at the core outlet is caused by the initial development of natural circulation in which the pressure loss of the primary transport system plays an important role. On the other hands, it can be concluded that the heat transfer coefficient has almost no influence on the output uncertainty. Since liquid sodium has a high thermal conductivity, the variation of the heat transfer coefficient results in a small temperature difference between the structure and the working fluid.

With regard to the thirdly local maximum that is investigated in the long period of time, the heat transfer coefficient at the ACS (air cooler) is the most dominant parameter. Since the decay heat of the core is removed finally to atmospheric via the ACS. Therefore, the capability of the heat removal in the ACS is of importance. Hence the heat transfer coefficient especially at the air side be a key issue for the output uncertainty.

As concerns the influence of code runs on the quantification of the input variables, almost the similar tendency is obtained when a predominant factor is precise as seen in Fig. 5 b). On the contrary, the correlation ratio varies a little bit in case of the short period although the general tendency seems to be similar. In the present study, 18 input parameters are changed simultaneously. It can be said that a multiple-stage usage of the present method will be more efficient to investigate the correlation ratio precisely with a small number of code runs.



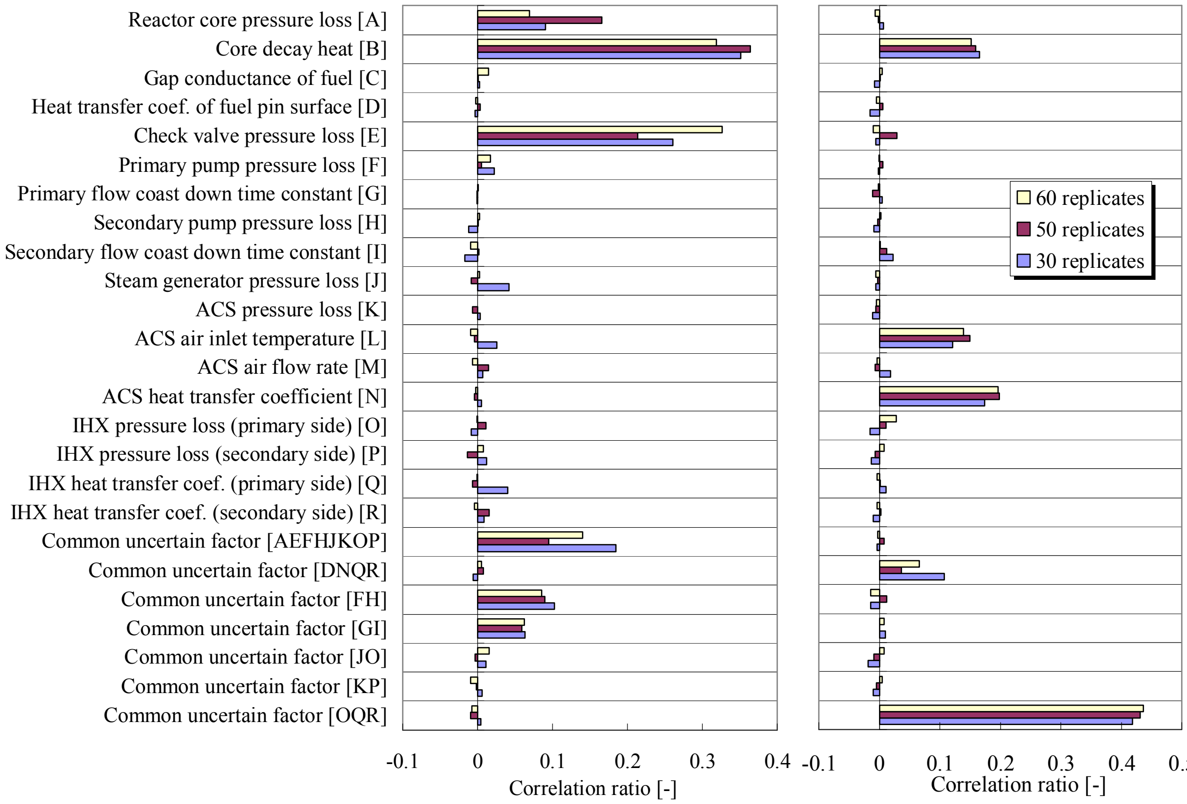
a) Short period (0-100s) **b) Long period (0-24hr)**

Fig. 5: Correlation ratio without common uncertain factor

Figure 6 shows the comparison of the correlation ratio considering the common uncertain factors. The capital alphabets in the common uncertain factors reveal the grouping of the input parameters (see Table 2).

In case of the secondary local maximum of the coolant temperature (short period of time), the similar result is investigated as seen in Figs. 5 a) and 6 a). When the common uncertain factors are considered, the influence of the pressure loss coefficient is segmented into the independent part and the common part. Accordingly, the correlation ratios of the pressure loss coefficient decrease and the correlation ratio of the core decay heat increases relatively.

As shown in Fig. 6 b), the correlation ratio of the common uncertain factor [OQR] that corresponds to the heat transfer coefficient common part among the IHX and the ACS based on the same empirical heat transfer correlation becomes the most dominant effect toward the output uncertainty. Consequently, it can be concluded that the uncertainty of the heat transfer coefficient will be most important to the thirdly local maximum coolant temperature.



a) Short period (0-100s) **b) Long period (0-24hr)**

Fig. 6: Correlation ratio with common uncertain factor

4. CONCLUSION

A stochastic analysis has been carried out based on the Midpoint Latin Hypercube Sampling (MLHS). As a quantitative index, the correlation ratio is selected. The natural circulation decay heat removal in sodium cooled fast reactor is chosen as a target phenomenon and the secondary local maximum of the coolant temperature at the core outlet is selected as the code output for the short period (0-100s) phenomenon as well as the thirdly local maximum of the coolant temperature at the upper plenum as for the long period (0-24hr) of the phenomenon.

In the analysis, 18 input variables are chosen as the input uncertainty. Then each input variables is segmented into 10 strata of equal marginal probability and 30-60 replicates have been examined, which corresponds to 300-600 code runs respectively.

In the random sampling of the input variables, the common uncertain factor, which comes from a common background of the uncertainty such as an empirical correlation and working fluid, has been introduce.

As a result of the numerical investigation, it is demonstrated that the uncertainty of the pressure loss coefficient in the check valve affects the output uncertainty most, followed by the core decay heat, the pressure loss coefficient of the primary pump and the reactor core in case of the short period. This is attributed the fact that the secondary local maximum of the coolant temperature that appears at that period is caused by the initial development of natural circulation especially at the primary heat transport system. Since the thirdly local maximum of the coolant temperature is affected by the global energy balance between the decay heat and the heat removal to atmospheric air, the heat transfer coefficient of the air cooler (ACS) plays an important role when no common uncertain factor is taken into account.

When one considers the common uncertain effect, the common factor of the heat transfer coefficient at the intermediate heat exchanger (IHX) and the ACS is of importance in the long period of the phenomenon, although an obvious change of the correlation ratio is not investigated in the short period phenomenon.

It is also demonstrated that the influence of code runs seems to be negligible when a predominant factor is precise. On the other hands,, the correlation ratio varies a little bit in case of the short period although the general tendency seems to be similar. It can be said that a multiple-stage usage of the present method will be more efficient to investigate the correlation ratio precisely with a small number of code runs when many input variables are taken into account at the same time.

It is concluded that the present method is promising to investigate the influence of the input uncertainty on the code output quantitatively even if the input variables are connected nonlinearly. Concurrently, a mature discussion will be required so as to determine the statistic properties of each input variables and the common uncertain factor among the input variables.

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