α-Decomposition Method: A New Approach to the Analysis of Common Cause Failure

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Abstract: Traditional α-factor model has focused on the occurring frequencies of common cause failures (CCF) events and α-factors are defined by the failure data of groups of components. On the other hand, one global α-factor is actually determined by a set of possible causes, which are characterized by various risk significances. In the present paper, the global α-factor is distinguished based on risk significances of all constituent elements (α-decomposition) and a new methodology is developed to estimate it using Bayesian approaches. Firstly, we regard that the upcoming CCF is directly caused by the appearance of some potential causes and adopt a Bayesian network to reveal a relationship of all potential causes and possible failures. Secondly, because all potential causes have different occurring frequencies and various abilities to trigger dependent failures or independent failures, we have developed a mathematical model to analyze the global α-factor by expressing the correlations among global α-factors, causes' occurring frequencies and CCF-triggering abilities (α-factors of causes). At last, we use an example inferred by means of Bayesian approaches to evaluate CCF risk and to update the distribution of global α-factors based on a database and new occurring evidence of specific causes. It is demonstrated that the new approach has the ability to find the most hazardous causes, and that it is convenient to apply and more effective to express the affection of new evidence, as a result of combining α-factor model with Bayesian approaches.

Keywords: Common cause failure, α-factor model, α-decomposition, Bayesian theory

1. INTRODUCTION

As a conclusion from probabilistic safety analysis (PSA) of nuclear power plants (NPPs), common cause failures (CCFs) are significant challenges to the availability of safety systems, whose reliability is strengthened by increasing the redundancy in the system design. According to the point estimates, NUREG/CR-4780 [1] and NUREG/CR-5458 [2] provided several basic principles, models and guidance to analysts performing CCF analysis, e.g. β-factor model, Multi Greek Letter, α-factor model, and Binomial Failure Rate, etc.. Along with the development of CCF modeling methodology, CCF databases have been developed by United States Nuclear Regulatory Commission (U.S. NRC) in these years [3]. Furthermore, since 21st century, the emphasis of CCF analysis has converted from simple mathematical models to more complicated event assessment and causal inference. Based on U.S. commercial NPPs events data, NUREG/CR-6819 [4] illustrated further understanding of CCF insights for emergency diesel generators, motor-operated valves, pumps and circuit breakers. Updated version of NUREG/CR-6268 [5] presented the process of event data collection and grouped the hierarchy of proximate failure causes, which obviously provided a way to gain further understanding of the CCF events' occurring. Rasmuson et al. extended the previous work on the treatment of CCF in event assessment [6]. Fully expanded fault trees were used, which show all terms in the basic parameter model (BPM), respectively. Kelly et al. proposed the preliminary framework of a causality-based model via Bayesian networks which has the potential to overcome limitations of BPM [7]. These methods aimed to tell the retrospective conditional failure probability of remaining equipments, given observed equipment failures and associated causes, and also aimed to provide cause-specific quantitative insights into likely causes of failures.

As one family of graphical representation of distributions, Bayesian network uses a directed graph to represent a set of independencies and to factorize a distribution, which can promote the visualized analysis of CCF [8][9]. Besides, Bayesian statistical inference provides a way of formalizing the process of learning from data to update beliefs in accord with recent notions of knowledge synthesis [10]. Based on frequentist probability, the widely applied BPM enables the uncomplicated evaluation of CCF probability, but it has limitations to identify the specific potential causes and is unable to forecast the posterior distribution under the condition of data-missing problem as well as to combine new obtained evidence that predicts the risk of CCF. However, if we abandon the lumped BPM, the pure Bayesian network means incredibly huge calculation, so it is important to combine the lumped basic parameters and causal inference together. In this paper, authors propose a new approach to analyze CCF, which is named α-decomposition method: (1) based on event insights of CCF, global α-factors are decomposed since every possible cause has different risk significance to CCF; (2) Bayesian inference is applied to update the prior distribution of parameters as a result of the appearance of new evidence.

2. α-FACTOR MODEL FOR STANDARD CCF ANALYSIS

Before the introduction of α-decomposition method, for the purpose of easy understanding of the notation, a review of standard α-factor model is necessary. For instance, here let us consider a system of three identical components *A*, *B*, and *C*, with a two-out-of-three success logic, so the common-cause component group (CCCG) is *A*, *B*, and *C*. There, a group of components identified in the process of CCF analysis is called as common cause component group. The minimal cutsets of this system failure are:

$$\{A, B\};\ \{A, C\};\ \{B, C\};\ \{A, B, C\}.$$

The cutsets of component *A* failure are:

$$\{A_I\};\ \{C_{AB}\};\ \{C_{AC}\};\ \{C_{ABC}\}.$$

Where, A_I: failures of component A from independent causes; C_{AB}: failures of components A and B from common causes; C_{AC}: failures of components *A* and *C* from common causes; C_{ABC}: failures of components *A*, *B* and *C* from common causes. Using the rare event approximation system failure probability, the system failure probability of the two-out-of-three system is given by:

$$P(S) \cong P(A_I)P(B_I) + P(A_I)P(C_I) + P(B_I)P(C_I) + P(C_{AB}) + P(C_{AC}) + P(C_{BC}) + P(C_{ABC}) \quad (1)$$

The failure probability of component *A* is decomposed as:

$$P(A_t) = P(A_I) + P(C_{AB}) + P(C_{AC}) + P(C_{ABC}) \quad (2)$$

Where, A_t: all failures of component; $P_{(X)}$: probability of event *X*. And assume that:

$$\begin{gathered} P(A_I) = P(B_I) = P(C_I) = Q_1 \\ P(C_{AB}) = P(C_{AC}) = P(C_{BC}) = Q_2 \\ P(C_{ABC}) = Q_3 \end{gathered} \quad (3)$$

So the system failure probability is

$$Q_s \cong 3(Q_1)^2 + 3Q_2 + Q_3 \quad (4)$$

And the component *A* failure probability is

$$Q_t = Q_1 + 2Q_2 + Q_3 \quad (5)$$

In this paper, the system is assumed as staggered testing scheme. The definition of α-factors (staggered testing scheme):

$$\alpha_1 = \frac{Q_1}{Q_t};\ \alpha_2 = \frac{2Q_2}{Q_t};\ \alpha_3 = \frac{Q_3}{Q_t} \quad (6)$$

3. PROPOSED α-DECOMPOSITION METHOD WITH HYBRID BAYESIAN NETWORK (HBN)

3.1. The Definition of α-decomposition

Based on the α-factor method, we have already illustrated how to do the current CCF analysis modeling. The α-factors only express how much are the probabilities of the independent failure part and the CCF part, and actually it is difficult to find the origin of CCF and also difficult to find how to reduce the CCF part. On the purpose of finding and ranking the causes with CCF risk significance, we would propose the α-decomposition method. In this paper, the α-factors in components' level are named as global α-factors, and after decomposition, α-factors in causes' level are named as decomposed α-factors.

Definition 1 (α-decomposition):
Decompose or factorize the global α-factor from standard CCF analysis model (α-factor model) according to different CCF triggering significance of every cause, by applying the Bayesian network to represent the relationship of global α-factors and decomposed α-factors.

When we do the specific cause analysis of CCF event, different causes or initial events have different abilities to trigger a CCF. For instance, as one of examples depicted in NUREG/CR-6819, 36% of all CCFs are caused by design, etc., and 7.3% by external environment. At the same time, the same kind of causes with different levels of magnitude will have different abilities to cause a CCF. According to the research of seismic probabilistic safety assessment (SPSA), in the low peak ground acceleration (PGA) range, the β-factor is quite small, but in the high PGA range the β-factor is quite large [11]. For another example, the risk of CCFs being caused by design is that either design errors frequently happen or design errors are good at CCF triggering. Obviously, when two kinds of seismic happen, the higher PGA tends to generate CCF much easier than the lower PGA.

One other important failure-triggering characteristic of a cause is that when a cause happens, it might generate a CCF of multiple components or it might generate only an independent failure. In α-factor model the independent part is expressed as α1 and multiple failures are expressed as $\alpha_2, \alpha_3, \ldots, \alpha_n$. For example, in the analysis of SPSA, even the common cause seismic event sometimes causes only independent failure, so the β is always less than 1, and also in the analysis of design error, it is obvious that also not all of design error will result in CCF. From those examples, we could judge that there are two possible characteristics of causes that lead to different fraction of CCF occurring. One is the CCF-triggering ability of a kind of causes, and the other is the occurring frequency of each cause. Therefore, it is possible to decompose the α-factor and find the most hazardous causes which happen frequently with great CCF triggering ability.

3.2. The HBN for α-decomposition Method

The global α-factor could be treated as a joint distribution. Traditionally, it is represented by a joint density function, a curve or even by a summary of distribution (percentiles, mean, and median, etc.). In this paper, two kinds of relationships are considered: the relationship of system failure and α-factors; that of α-factors and causes, thus, we apply hybrid Bayesian network (HBN) to express both of the relationships. A hybrid network means to use a combination of two or more topologies, and here the HBN is a combination of Bayesian network and Fault Tree (FT). Let's consider a system of three components *A*, *B*, and *C*. In standard α-factor model, all components of a system are assumed identically, but for α-decomposition method, it is also available for components that are partially identical. For the simplest consideration, we assume that all of three components, *A*, *B*, and *C* are identical and that there are three potential causes, C_1, C_2, and C_3 that will possibly lead to the component failures (as shown in Fig.1).

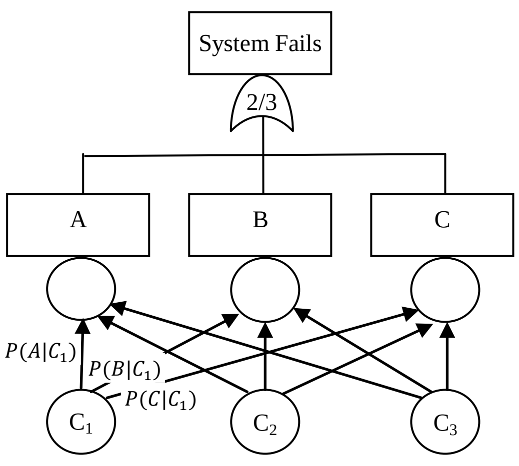

Fig.1 Hybrid Bayesian network for CCF analysis

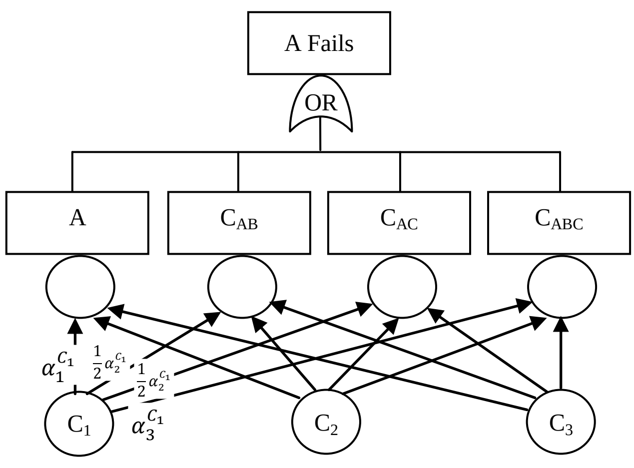

Fig.2 α-decomposition for CCF analysis with HBN

The conditional probabilities of component failure and system failure are expressed in Fig.1. Take component *A* as an example:

$$P(A) = P(A|C_1)P(C_1) + P(A|C_2)P(C_2) + P(A|C_3)P(C_3) \quad (7)$$

The process of α-decomposition method is illustrated in Fig.2, and the mechanism of *Cause i* (*i*=1,2,3) is operated by generating independent *A* failure; *A* and *B* failures; *A* and *C* failures; *A*, *B* and *C* failures:

$$P(A|C_i) = P(A_I|C_i) + P(AB|C_i) + P(AC|C_i) + P(ABC|C_i) \quad (8)$$

Both sides of equation are divided by $P(A|C_i)$:

$$1 = \alpha_1^{C_i} + \alpha_2^{C_i} + \alpha_3^{C_i} \quad (9)$$

Where,

$$\alpha_1^{C_i} = \frac{P(A_I|C_i)}{P(A|C_i)};\ \alpha_2^{C_i} = \frac{P(AB|C_i)}{P(A|C_i)} + \frac{P(AC|C_i)}{P(A|C_i)};\ \alpha_3^{C_i} = \frac{P(ABC|C_i)}{P(A|C_i)} \quad (10)$$

For the purpose of distinguishing the difference between two kinds of α-factors, in this paper, we named the α_j $(j = 1,2,3)$ as global α-factors, and named the $\alpha_j^{C_i}$ $(i,j = 1,2,3)$ as causes' α-factors. The independent failure of component is affected by *Cause 1*, *Cause 2* and *Cause 3*, and according to Fig.2, the relationship is expressed as:

$$P(A_I) = P(A_I|C_1)P(C_1) + P(A_I|C_2)P(C_2) + P(A_I|C_3)P(C_3) \quad (11)$$

Replace the independent part of probability with $\alpha_1^{C_i}$ elements as follows,

$$P(A_I) = \alpha_1^{C_1}P(A|C_1)P(C_1) + \alpha_1^{C_2}P(A|C_2)P(C_2) + \alpha_1^{C_3}P(A|C_3)P(C_3) \quad (12)$$

Both sides of equation are divided by $P(A)$ and because $\frac{P(A_I)}{P(A)} = \alpha_1$, so we get as follows,

$$\alpha_1 = \alpha_1^{C_1}\frac{P(A|C_1)P(C_1)}{P(A)} + \alpha_1^{C_2}\frac{P(A|C_2)P(C_2)}{P(A)} + \alpha_1^{C_3}\frac{P(A|C_3)P(C_3)}{P(A)} \quad (13)$$

Deduced from Fig.1, it is known that failure of *A* is generated by three parts, and we notate these three parts as fractions or rates:

$$r_i = \frac{P(A|C_i)P(C_i)}{P(A)} = P(C_i|A); i = \{1,2,3\} \quad (14)$$

Where, $r_1 + r_2 + r_3 = 1$. So we get very simple form of α_1-decomposition:

$$\alpha_1 = \alpha_1^{C_1}r_1 + \alpha_1^{C_2}r_2 + \alpha_1^{C_3}r_3 \quad (15)$$

And also we can obtain the simple forms of α_2-decomposition and α_3-decomposition:

$$\begin{aligned} \alpha_2 &= \alpha_2^{C_1}r_1 + \alpha_2^{C_2}r_2 + \alpha_2^{C_3}r_3 \\ \alpha_3 &= \alpha_3^{C_1}r_1 + \alpha_3^{C_2}r_2 + \alpha_3^{C_3}r_3 \end{aligned} \quad (16)$$

Where, α_j $(j = 1,2,3)$: global α-factors for j components failure; $\alpha_j^{C_i}$ $(i,j = 1,2,3)$: decomposed α-factors for j components failure as a result of *Cause i*; r_i $(i = 1,2,3)$: the occurrence fraction for *Cause i*.

The $\alpha_j^{C_i}, r_i (i,j = 1,2,3)$ have practical engineering meanings: The $\alpha_j^{C_i}$means the ability of *Cause i* to lead to the j components failure; the r_i means that among all failures, totally there are $r_i \times 100\%$ failures generated by *Cause i*. In other words, r_i is the occurrence fraction over the occurrence of all effective causes. (Effective cause means the cause really triggers a failure whether independent or dependent and oppositely, if a cause happens but no failure is generated, it is not an effective cause.) For example, if *Cause 1* happens much more frequently than other causes, the global α_j will tend to be $\alpha_j^{C_1}$*;* on the other side, if *Cause 2* happens rarely with low $\alpha_j^{C_2}$, it means that risk-significance of *Cause 2* is negligible.

4. BAYESIAN INFERENCE FOR α-DECOMPOSITION METHOD

4.1. Stochastic Modeling and Bayesian Theorem

The development and implementation of Markov Chain Monte Carlo (MCMC) methods make the Bayesian statistics practically available to complicated models. Markov chains are process describing trajectories where successive quantities are described probabilistically according to the value of their immediate predecessors, and MCMC techniques enable simulation from a distribution by embedding it as a limiting distribution of a Markov chain and simulating from the chain until it approaches equilibrium [12][13]. For the analysis of CCF, prior distributions may only roughly represent CCF beliefs, but it is broadly useful for statistical inference. In combination with the new evidence and experts' opinions, the posterior distributions of CCF parameters would provide analysts more accurate understanding of risk significance.

In last section, we have already proved the relationship between global α-factors (α_j $(j = 1,2,3)$) and decomposed α-factors ($\alpha_j^{C_i}$ $(i, j = 1,2,3)$), which is named as α-decomposition process.

$$\alpha_j = \alpha_j^{C_1} r_1 + \alpha_j^{C_2} r_2 + \alpha_j^{C_3} r_3 \tag{17}$$

From the viewpoint of statistical modeling, this model is a collection of probabilistic statements that is able to describe and interpret present behavior and predict future performance. It consists of one response variable α_j, three explanatory variables r_i $(i = 1,2,3)$, and three parameters $\alpha_j^{C_i}$ $(i = 1,2,3)$, and these three parameters $\alpha_j^{C_i}$ $(i = 1,2,3)$ are actually a linking mechanism between response variable and explanatory variables.

The response variables (global α-factors) are the main study variables, and they represent the stochastic part of the model. Our interest is to evaluate the mechanism between response variables and explanatory variables, and also to predict a future outcome of response variables. For this model, we can also write:

$$\alpha_j | r_1, r_2, r_3 \sim \mathcal{D}(\boldsymbol{\theta}) \tag{18}$$

Where, $\mathcal{D}(\boldsymbol{\theta})$ is a distribution with parameter vector $\boldsymbol{\theta}$, and the signal "~" means the stochastic form of global α-factor α_j given explanatory variables r_i $(i = 1,2,3)$ is $\mathcal{D}(\boldsymbol{\theta})$.

For simplest consideration, the response is assumed as truncated normal distribution and the response can be written as a truncated normal regression model:

$$\alpha_j | r_1, r_2, r_3 \sim N\left(\mu_j\left(\alpha_j^{C_1}, \alpha_j^{C_2}, \alpha_j^{C_3}, r_1, r_2, r_3\right), \sigma^2\right) \tag{19}$$

Where $N(\mu_j, \sigma^2)$ is the truncated normal distribution with mean μ_j and variance σ^2, and

$$\mu_j\left(\alpha_j^{C_1}, \alpha_j^{C_2}, \alpha_j^{C_3}, r_1, r_2, r_3\right) = \sum_{i=1}^{3} \alpha_j^{C_i} r_3 \tag{20}$$

Because there is a Sum-to-One constraint for global α-factors, therefore,

$$\alpha_1 + \alpha_2 + \alpha_3 = 1 \tag{21}$$

Furthermore, for *Cause i's* decomposed α-factors:

$$\alpha_1^{C_i} + \alpha_2^{C_i} + \alpha_3^{C_i} = 1 \tag{22}$$

Thus, we can estimate the $\alpha_1^{C_i}$ based on the result of $\alpha_2^{C_i}$ and $\alpha_3^{C_i}$:

$$\alpha_1^{C_i} = 1 - \alpha_2^{C_i} - \alpha_3^{C_i} \quad (23)$$

Bayes' theorem provides an expression for the conditional probability of global α-factors and causes' α-factors given the rates of causes' occurrence, which is shown as:

$$P\left(\alpha_j^{C_i}\middle|\boldsymbol{r}, \alpha_j\right) = \frac{P\left(r, \alpha_j\middle|\alpha_j^{C_i}\right)P(\alpha_j^{C_i})}{P(r, \alpha_j)} \quad (24)$$

In equation (24), $P(\alpha_j^{C_i})$ could be defined as prior distribution of decomposed α-factors, and $P\left(\boldsymbol{r}, \alpha_j\middle|\alpha_j^{C_i}\right)$ could be defined as the likelihood of parameters. Therefore, the posterior distribution can be written as

$$f\left(\alpha_j^{C_i}\middle|\boldsymbol{r}, \alpha_j\right) = \frac{f\left(r, \alpha_j\middle|\alpha_j^{C_i}\right)f(\alpha_j^{C_i})}{f(r, \alpha_j)} \propto f\left(\boldsymbol{r}, \alpha_j\middle|\alpha_j^{C_i}\right)f(\alpha_j^{C_i}) \quad (25)$$

So the posterior distribution is explained by the prior distribution $f\left(\alpha_j^{C_i}\right)$ and the likelihood function $f\left(\boldsymbol{r}, \alpha_j\middle|\alpha_j^{C_i}\right)$. Where, α_j : the global α-factor for j components' failure; $\alpha_j^{C_i}$: the *Cause i's* α-factor for j components' failure; $\boldsymbol{r}$: the set of parameters representing each cause's occurrence rate.

4.2. An Example for α-decomposition's Bayesian Inference

The following example is proposed by authors to explain how to do Bayesian inference for α-decomposition method in the analysis of CCF analysis. We would like to caution that all the CCF data used in this paper is for illustration only and not from any real database. All the calculation of Bayesian inference is conducted by WinBUGS version 1.4.3 and R version 2.13.1. A hypothetical database is assumed for the occurrence of causes and global α-factors. CCF data of 16 systems are supposed, including the occurrence times of every cause and the record of single failure (1/3), partially dependent failure (2/3), and complete failure (3/3). It is remarked here that all the recorded causes triggered failures.

$$P(Failure|Cause) = 1 \quad (26)$$

$$Total = \sum_{j=1}^{3} Cause\ j's\ occurrence\ rate = Partial\ CCF + Complete\ CCF \quad (27)$$

Additionally, it is assumed that all the occurrences of causes are independent.

$$P(Cause\ 2|Cause\ 1) = P(Cause\ 2) \quad (28)$$

When the databases are being built, all causes that have triggered CCF should be recorded and should not be recorded repeatedly. The cause of every CCF needs to be well defined. According to such assumptions and the requirement of analysis, a hypothetical database is assumed with available global α-factors and causes recording. Common causes' occurrence rate and global α-factors can be obtained as shown in Table 1. For example, for the recording of system Pump Segment (Pump), there are three possible causes for CCF, and the occurrence rates of causes are different, which are 16.00%, 73.60%, and 10.40%, respectively. Currently there are such research work conducted by U.S. NRC and the occurrences of proximate causes are classified and recorded [4][5]. Because of the lack of space, the calculation of occurrence rate and the description of CCF events database are omitted in this paper. Besides, global α-factors are calculated based on equations (6) which can be obtained from CCF parameters databases [3], and the global α-factors for system Pump Segment (Pump) are 9.21E-01, 6.97E-02, and 8.90E-03, respectively. Thus, in the hypothetical database, CCF causes recording and CCF parameter estimation are recommended to be combined together in order to conduct α-decomposition analysis.

Table 1 Parameters applied in α-decomposition method

Four systems for CCF analysis	**Common causes' occurrence rate**			**Global α-factors**		
	r_1	r_2	r_3	α_1	α_2	α_3
Pump Segment (Pump)	16.00%	73.60%	10.40%	9.21E-01	6.97E-02	8.90E-03
Pump Segment (Driver)	20.70%	21.80%	57.50%	8.46E-01	9.94E-02	5.50E-02
Pump Segment (Suction)	43.90%	9.10%	47.00%	8.00E-01	8.18E-02	1.18E-01
Pump Segment (Discharge)	20.00%	33.33%	46.67%	8.63E-01	9.17E-02	4.58E-02
Circuit Breakers (RPS Trip Breakers)	14.00%	66.00%	20.00%	9.16E-01	7.76E-02	6.75E-03
Circuit Breakers (Medium Voltage)	35.30%	44.10%	20.60%	9.29E-01	7.05E-02	0.00E+00
Circuit Breakers (480 Vac Circuit Breakers)	6.50%	71.00%	22.50%	9.16E-01	8.11E-02	2.70E-03
Motor-Operated Valve (Actuator)	25.20%	22.00%	52.80%	8.86E-01	9.42E-02	1.97E-02
Motor-Operated Valve (Valve)	31.80%	18.20%	50.00%	8.97E-01	8.86E-02	1.40E-02
EDG (Instrumentation and Control)	36.60%	22.00%	41.40%	7.89E-01	8.24E-02	1.29E-01
EDG (Engine)	47.60%	38.10%	14.30%	9.31E-01	5.28E-02	1.65E-02
EDG (Fuel Oil)	15.80%	31.60%	52.60%	8.30E-01	9.84E-02	7.16E-02
EDG (Generator)	43.80%	18.80%	37.40%	8.74E-01	7.11E-02	5.53E-02
EDG (Cooling)	33.30%	20.00%	46.70%	7.86E-01	8.55E-02	1.28E-01
EDG (Starting Air)	36.40%	45.50%	18.10%	8.81E-01	6.47E-02	5.39E-02
EDG (Output Circuit Breaker)	11.10%	66.70%	22.20%	8.57E-01	7.93E-02	6.34E-02

Therefore, based on $[r_1,\ r_2,\ r_3]^T$, $[\alpha_1,\ \alpha_2,\ \alpha_3]^T$ and α-decomposition method, we are able to evaluate the distribution of decomposed α-factors and typical distributions of global α-factors. Because α_1 can be substituted by α_2 and α_3, as well as that α_1 represents the independent part the failure, in the following we only analyze the result of α_2 and α_3. For simplest consideration, the response variables (α_2, α_3) are assumed as truncated normal distribution $(0 \leq \alpha_j \leq 1, j = 2{,}3)$ with mean (μ_2, μ_3) and variance σ_2^2, σ_3^2.

$$\alpha_j \sim Normal\left(\mu_j, \sigma_2^2\right) \tag{29}$$

Prior distributions of decomposed α-factors are assumed as lognormal distribution:

$$Prior\ \alpha_j^{C_i} \sim Lognormal(0.01{,}0.1) \tag{30}$$

On the basis of equation (16) and (19), the parameters of global α-factors can be expressed with decomposed α-factors as well as occurrence rate:

$$\mu_j = \sum_{i=1}^{3} \alpha_j^{C_i} \times r_i \tag{31}$$

The prior distributions of global α-factors' variance are assumed as

$$Prior\ \sigma_j^2 \sim Gamma(0.01{,}0.01) \tag{32}$$

Thus, the likelihood of explanatory variables r_i and α_j can be calculated based on the prior distributions of parameters $\alpha_j^{C_i}$, and then the marginal posterior distribution of $\alpha_j^{C_i}$ can be obtained as

$$Posterior\ \alpha_j^{C_i} \propto Prior\ \alpha_j^{C_i} \times Likelihood\left(r_i, \alpha_j \middle| \alpha_j^{C_i}\right) \tag{33}$$

With the defining of prior distributions of variables (30) ~ (33) and stochastic model of global α-factors (17), the calculation of posterior distributions is conducted by Bayesian inference (33) with Markov Chain Monte Carlo (MCMC) Gibbs Sampling. The summaries of posterior PDF for decomposed α-factors are shown in Table 2. It could be observed from Table 2 that even though all prior distributions for decomposed α-factors are assumed same, results for posterior distribution vary from different causes. It is obvious that causes have different ability to trigger a CCF event, so the posterior mean and standard deviation of decomposed α-factors are different from each other.

Table 2 Summary of posterior distributions after Bayesian inference

Posterior $\alpha_j^{C_i}, \alpha_j\ (i,j=2,3)$		**Mean**	**SD**	**2.5%**	**Median**	**97.5%**
α_2-decomposition	$\alpha_2^{C_1}$	4.39E-02	4.35E-02	5.79E-04	3.03E-02	1.55E-01
	$\alpha_2^{C_2}$	6.04E-02	3.24E-02	3.52E-03	5.96E-02	1.27E-01
	$\alpha_2^{C_3}$	1.25E-01	4.49E-02	2.22E-02	1.29E-01	2.05E-01
	Typical α_2	7.85E-02	1.04E-02	5.80E-02	7.88E-02	9.88E-02
α_3-decomposition	$\alpha_3^{C_1}$	6.33E-02	5.48E-02	6.82E-04	4.97E-02	1.92E-01
	$\alpha_3^{C_2}$	1.80E-02	1.99E-02	1.87E-04	1.09E-02	7.17E-02
	$\alpha_3^{C_3}$	7.53E-02	4.84E-02	1.74E-03	7.36E-02	1.71E-01
	Typical α_3	5.05E-02	1.36E-02	2.34E-02	5.05E-02	7.74E-02

The specific posterior probability density functions of $\alpha_2^{C_i}$ and $\alpha_3^{C_i}$ $(i=1,2,3)$ are shown in Fig.3 and Fig.5, respectively, and also cumulative density functions of $\alpha_2^{C_i}$ and $\alpha_3^{C_i}$ $(i=1,2,3)$ are shown in Fig.4 and Fig.6, respectively. From these pictures, *Cause 3* is the cause of the most serious risk-significance for CCF as a result of the greatest value. Compared with *Cause 2*, *Cause* 1 is better at creating a complete CCF but worse at creating a partial CCF. So that, if measures are taken for defend CCF and the same occurrence rates are reduced, there would result in different reduction of global α-factors. For example, here the defense against *Cause 3* is relatively more efficient to reduce global α-factors. However, the variance of $\alpha_j^{C_1}$ and $\alpha_j^{C_3}$ are comparatively larger than $\alpha_j^{C_2}$, which mean is *Cause 1* and *Cause 3* have wider CCF risk-significance domain than that of *Cause 2*. In other words, *Cause 2*'s CCF triggering ability varies in a narrow domain and *Cause 2* has a more typical value of risk-significance. In contrast to *Cause 2*, *Cause 1* and *Cause 3's* risk-significance fluctuates in a wider range, which means that both of them have relatively various magnitudes. Therefore, when causes happen and if causes' information is very limited, it is more difficult for *Cause 1* and *Cause 3* to decide what kind of the risk-level they will bring about. And approximately, the typical form of global α-factors can be obtained from the summaries of decomposed α-factors:

$$\alpha_j = \overline{r_1}\alpha_j^{C_1} + \overline{r_2}\alpha_j^{C_2} + \overline{r_3}\alpha_j^{C_3} \tag{34}$$

Where, $\overline{r_i}$ represents the mean value of occurrence rates for *Cause i*. From Table 2, the approximate means of *typical* α_2 and *typical* α_3 are 7.85E-02 and 5.05E-02, respectively. The probability density distributions of *typical* α_2 and *typical* α_3 are shown in Fig.7, which are similar to normal distributions as a result of the simplest assumption. In the real analysis, the distribution can be obtained through the analysis of real database or expert opinion. Here, the mean value of *typical* α_2 is greater than that of *typical* α_3, and *typical* α_2 are more centered than *typical* α_3. It means that for those systems, partial CCFs are more common than complete CCFs, and certainly, it would be better if the global α_3 is extremely low, which means not all of components fail, and it is possible some of backups could be available.

According to the analysis of α-decomposition method, it is found that global α-factors could be considered as the weighted average of decomposed α-factors. This weighted average integrates the occurrence rate (r_i) and risk triggering ability of every cause ($\alpha_j^{C_i}$) together. It is an easy deduction that all of three causes have respective abilities to create a common cause failure. It would be a good method for improving the availability of our system if the occurrence of three causes could be avoided. But in practice, some common cases are inevitable. From the viewpoint of CCF occurring mechanism, these common cause events have to affect multiple components simultaneously, and the condition or mechanism through which failures of multiple components are coupled is termed as the coupling factor, which in other words, is a characteristic of a group of components that identifies them as susceptible to the same causal mechanism of failure. Therefore, based on the α-decomposition method, it is possible to recognize the most hazardous causes, and if specific measures could be taken to disrupt the coupling mechanism of multiple components, the occurrence probability of CCF events would be reduced. Because the jointed distributions of global α-factors only provide the probability of CCF events, it is difficult to know what is hazardous to the reliability of systems only based on global α-factors, and obviously, without the causes' analysis, it is difficult to know the detailed coupling factors of CCFs. The constitution of defense mechanism without enough information would become aimless. As a result of that, knowing the different risk significance of each cause would be helpful to establish a defense strategy against a CCF. Since generally there are two methods to defend a CCF: one is to defend against the failure cause, and the other is to defend against the CCF coupling factor, the understanding of potential causes and coupling factors is very important.

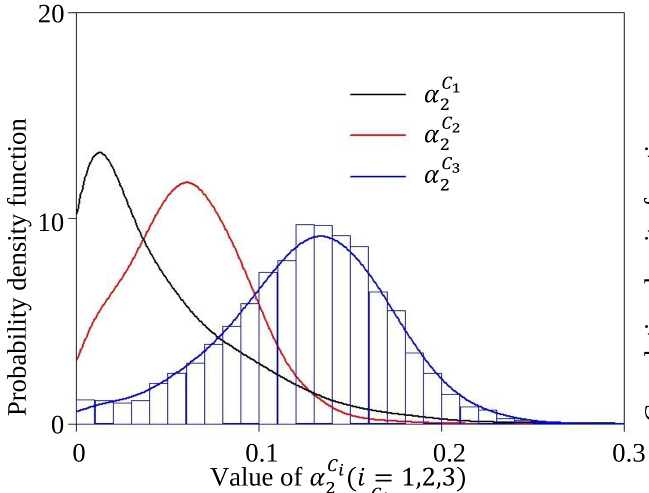

Fig.3 Posterior PDF of $\alpha_2^{C_i}$ $(i = 1,2,3)$

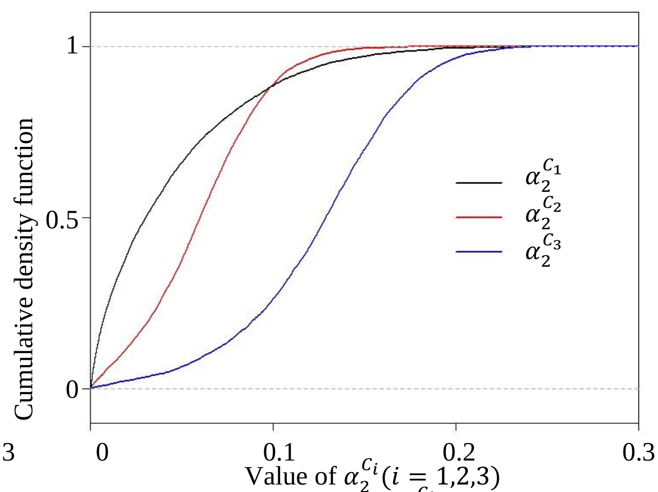

Fig.4 Posterior CDF of $\alpha_2^{C_i}$ $(i = 1,2,3)$

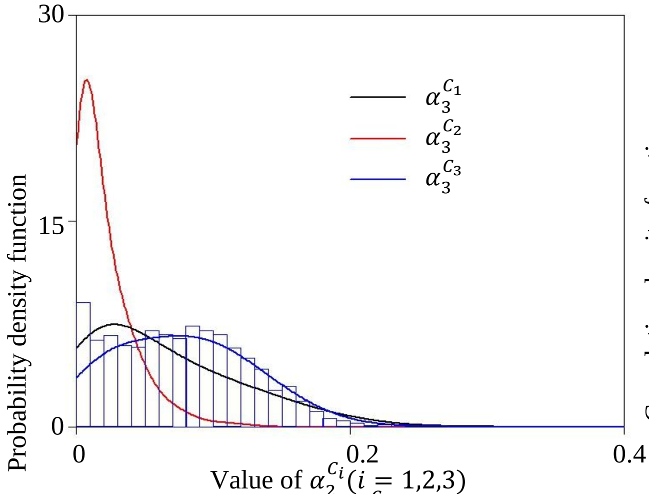

Fig.5 Posterior PDF of $\alpha_3^{C_i}$ $(i = 1,2,3)$

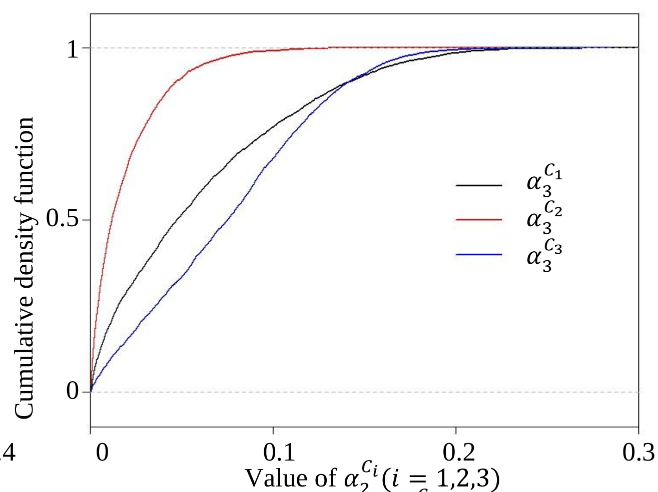

Fig.6 Posterior CDF of $\alpha_3^{C_i}$ $(i = 1,2,3)$

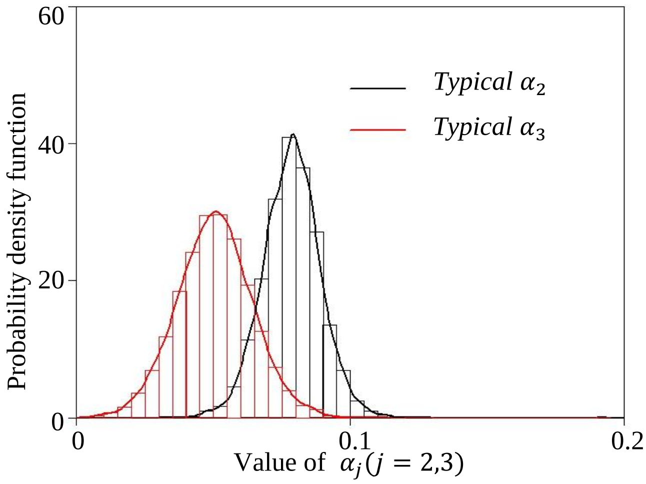

Fig.7 Posterior distributions of typical α_2 and typical α_3

5. CONCLUSION AND DISCUSSION

Based on α-factor modeling method, a new method has been introduced to analyze common cause failures in the aspect of causes' risk analysis, which is named as α-decomposition method. Global α-factors are integrated parameters, whose joint distributions combine all information generated by various causes. From the viewpoint of data analysis, global α-factors are averages of CCF occurrence frequencies during one certain period, which could be considered as weighted averages of all decomposed α-factors. Decomposed α-factors proposed in this paper are risk characteristics of specific varieties of causes, which represent different CCF triggering ability of potential causes. Such kind of analysis is useful to assist analysts to rank causes according to different risk significance to systems, and it is emphasized that the necessity of combining two currently independent databases to obtain posterior distribution of CCF parameters.

Firstly, for easy understanding of notation of α-decomposition method, the α-factor method is illustrated. Secondly, based on causes of different risk-significance, the CCF occurring mechanism is recognized as hierarchical network. The first level is system failure, the second level is components failure and the third level is possible causes. The theory of hybrid Bayesian network and conditional probability are applied to decide the correlation of the three levels. Thirdly, the form of CCF analysis database is explained, which represents the necessity of combination of CCF causes recording and CCF parameters databases. Lastly, based on this statistical model, the theory of Bayesian inference with MCMC method is introduced to evaluate all parameters of α-decomposition method. As an example, we illustrated the calculation procedure of α-decomposition method with a simple 3-component system using Bayesian inference and the calculation tools.

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