NTHAS8: The Eighth Korea-Japan Symposium on Nuclear Thermal Hydraulics and Safety
Beppu, Japan, December 9- 12, 2012

Paper Number N8P1091

QUANTIFICATION OF MODELING UNCERTAINTIES BASED ON SCALING LAWS IN NATURAL CIRCULATION DECAY HEAT REMOVAL

Keisuke Yamada[1*], Akira Yamaguchi[1] and Takashi Takata[1]

[1] Sustainable Energy and Environmental Engineering, Graduate School of Engineering Osaka University
2-1, Yamada-Oka, Suita, Osaka, 565-0871, JAPAN
phone: +81-6-6879-7891; e-mail: yamada_k@qe.see.eng.osaka-u.ac.jp

ABSTRACT

A Best Estimate Plus Uncertainty (BEPU) analysis is one of the good methods to estimate the uncertainty of phenomenon in a nuclear power plant dynamics. In BEPU analysis, a number of numerical analyses, in which input parameters are varied based on their probabilistic distributions, are carried out to obtain statistical characteristics of the output result. In general, the uncertainty of input parameters, such as a probabilistic distribution form and variance, are estimated based on experimental knowledge and/or engineering judgment. In the present research, we focus on a scaling law (dimensionless number) in constitutive equations from a view point of phenomenological theory. An influence of uncertainty in the dimensionless number and its dependency on BEPU analysis has been investigated. Plant dynamics analyses of Super Safe, Small and Simple (4S) reactor, being developed by Toshiba, are carried out under a natural circulation decay heat removal condition. In the analysis, uncertainties of the dimensionless numbers such as Nusselt, Reynolds, and Prandtl numbers are taken into consideration, as well as an uncertainty of decay heat power. The Latin Hypercube Sampling is applied to determine the input deck set. As a result, it is demonstrated that the parameter dependency on the output result can be revealed by using the dimensionless numbers.

1. INTRODUCTION

4S (Super-Safe, Small and Simple) reactor is one of the new type of Fast Reactor being developed by TOSHIBA. N.UEDA wrote about 4S reactor (N.UEDA, et al, (2003)). Liquid sodium is used as a coolant in this 4S reactor, and thus it has a passive safety system for decay heat removal using a natural circulation phenomenon. Since a driving force for the circulation will be added passively due to the temperature difference in case of the decay heat removal, no electricity supply is required to the system. Consequently, this system is quite useful when a Station Black Out (SBO) or loss of offsite power accident occur. However, natural circulation is very sensitive and has large uncertainties because a driving force generated from the buoyancy effect is very weak. Hence it is important to investigate an influence of the uncertainties on the plant dynamics.

A best estimate plus uncertainties (BEPU) simulation is an appropriate method to investigate such influences. In general, important input variables, such as the pressure drop and heat transfer coefficients, are evaluated based on scaling laws and thus dimensionless numbers are used in those correlations. Accordingly, uncertainties in the dimensionless numbers are more essential and will affect the input variables dependently. Therefore, BEPU analysis based on scaling laws, in which uncertainties of dimensionless numbers are considered, has been carried out in the present research. A plant dynamics simulation of natural circulation decay heat removal in 4S reactor is applied for this purpose. As a sampling manner and a quantification of the influence, the Latin Hypercube Sampling (LHS) and the correlation ratio, originally proposed by McKay (M. D. Mckay (1995)), are introduced respectively.

2. STOCHASTIC ANALYSIS METHOD

2.1 BEPU analysis

In BEPU analyses, a number of numerical analyses are carried out where input parameters are set randomly in accordance with their stochastic characteristics (uncertainties). Then one evaluates the probabilistic property of the output result. Figure 1 shows a schematic image of BEPU analysis. Where x_i is input parameters and y is the output result.

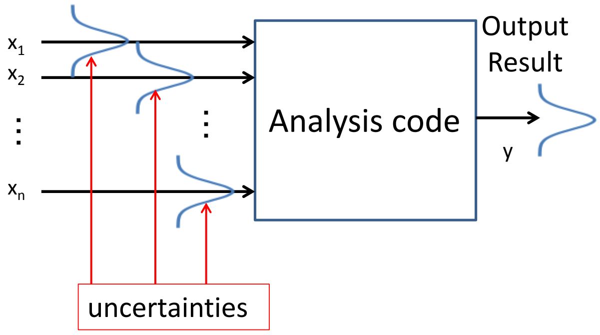

Fig.1 Conceptual diagram of BEPU analysis

2.2 Latin Hypercube Sampling

To input uncertainties into input parameters, Latin Hyper Cube Sampling (LHS) is used. The schematic of LHS is depicted in Figure 2.

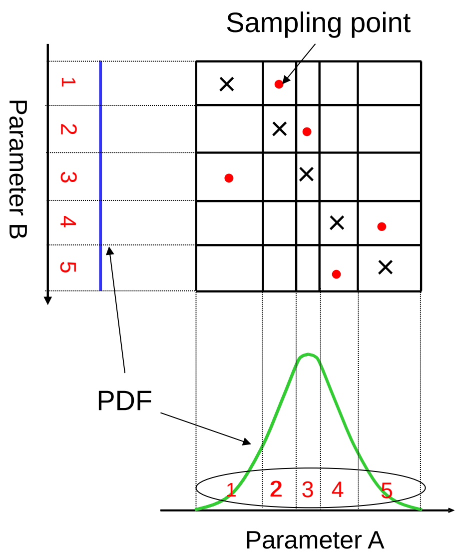

Fig.2 Schematic of LHS

For simplicity, only two input parameters are shown in Fig. 2. In the LHS, the Probabilistic Density Function (PDF) of each input parameter ($\mathbf{x}_s$) is divided into n strata of equal marginal probability (=1/n, n = 5 in Fig. 2). Next, a random sampling is done input variable set of the same level never appears in the LHS as a design matrix. Then additional random samplings are carried out to define each sampling point within the section of the equal marginal probability. The design matrix consists of $n \times s$. Here‘s’ is the number of input variables. Then the replicate of the matrix is generated based on the design matrix by applying the Latin square design where the cumulative stratum between the input variables does not appear in a same code run.

2.3 Correlation ratio

According to McKay (M. D. Mckay (1995)), the variance of code output (y) can be divided into two portions in the following.

$$V[y] = V[E(y\,|\,\mathbf{x}_s)] + E(V[y\,|\,\mathbf{x}_s]) \qquad (1)$$

Here, V[-] and E[-] indicate the variance and the expected value respectively. ‘**x**’ means the input variable vector and hence $V[y|\mathbf{x}_s]$ and $E(y|\mathbf{x}_s)$ denote the variance and expectation on condition that a subset $\mathbf{x}_s$ of the input vector is fixed. The first term of the right hand side of Eq. (1) is named as Variance of Conditional Expectation (VCE). The VCE means the magnitude of the correlation of the input parameter $\mathbf{x}_s$ and y is defined as:

$$V[E(y\,|\,\mathbf{x}_s)] = \int (E(y\,|\,\mathbf{x}_s) - E(y))^2 f_{x_s}(\mathbf{x}_s)dx_s \qquad (2)$$

Where f(-) denotes the Probability Density Function (PDF) of the variable.
The second term in the right hand of Eq. (1) means the within-group variance or the residual defined as:

$$E(V[y|\mathbf{x}_s]) = \int\int (y - E(y\,|\,\mathbf{x}_s))^2 f_{y|x_s}(y) f_{x_s}(\mathbf{x}_s)dydx_s \quad (3)$$

The residual reflects the variation of the individual data within the group from the group-mean value and thus it has nothing to do with $\mathbf{x}_s$. Accordingly, one can consider that Eq. (3) reveals the residual with respect to the uncertainty of $\mathbf{x}_s$.
Accordingly, the correlation ratio (η) that indicates the relative importance of the input parameter uncertainty with respect to the output uncertainty can be defined as a proportion of the VCE to the output variance V[y] as shown in the following.

$$\eta_{x_s} = \frac{V[E(y\,|\,\mathbf{x}_s)]}{V[y]} \qquad (4)$$

2.4 Discretization method

When one focuses on the input variable x_i, the sample average of the output (y) for $x_i = \mathbf{x}_{ij}$ is expressed as:

$$\bar{y}_j = \frac{1}{r}\sum_{k=1}^{r} y_{jk} \qquad (5)$$

Here r is the number of the replicate. Then the expected value of the variance of y_j is calculated as:

$$E(V\left[\bar{y}_j \middle| x_i\right]) = \frac{1}{n}\sum_{j=1}^{n}(\bar{y}_j - \bar{y})^2 \qquad (6)$$

where y means the average of all output. Equation (6) is rewritten in the following.

$$\begin{aligned} E(V\left[\bar{y}_j \middle| x_i\right]) &\approx V(E\left[\bar{y}_j \middle| x_i\right]) + E(V\left[\bar{y}_j \middle| x_i\right]) \\ &= V\left[E(y|x_i)\right] + \frac{1}{r}E(V\left[(y|x_i)\right]) \\ &= VCE(x_i) + \frac{1}{r}E(V\left[y|x_i\right]) \end{aligned} \qquad (7)$$

The expected value of the variance of y for which $x_i = x_{ij}$ is obtained as:

$$E(V\left[\bar{y}_j \middle| x_i\right]) = \frac{1}{nr}\sum_{j=1}^{n}\sum_{k=1}^{r}(y_{jk} - \bar{y}_j)^2 \qquad (8)$$

The VCE for the input variable x_i is then expressed by substituting Eqs. (5) and (7) into Eq. (6) as:

$$\begin{aligned} VCE(x_i) = &\frac{1}{n}\sum_{j=1}^{n}(\bar{y}_j - \bar{y})^2 \\ &- \frac{1}{nr^2}\sum_{j=1}^{n}\sum_{k=1}^{r}(y_{jk} - \bar{y}_j)^2 \end{aligned} \qquad (9)$$

The variance of the output is also obtained in the following.

$$V\left[y\right] = \frac{1}{nr}\sum_{j=1}^{n}\sum_{k=1}^{r}(y_{jk} - \bar{y}_j)^2 \qquad (10)$$

Finally, the correlation ratio as shown in Eq. (4) is discretized as:

$$\eta^2 = \frac{\frac{1}{n}\sum_{j=1}^{n}(\bar{y}_j - \bar{y})^2 - \frac{1}{nr^2}\sum_{j=1}^{n}\sum_{k=1}^{r}(y_{jk} - \bar{y}_j)^2}{\frac{1}{nr}\sum_{j=1}^{n}\sum_{k=1}^{r}(y_{jk} - \bar{y})^2} \qquad (11)$$

It is noted that the VCE must be positive value theoretically as shown in Eq. (4). However, it might be negative in the discrete manner as in Eq. (11). The negative vale of the VCE will appears when the focused input variable has no influence on the output.

3. UNCERTAINTY BASED ON SCALING LAW

Generally, each uncertainty of input parameters is independent in normal BEPU analysis. In such analysis, relationships of input parameters are not considered. However in fluid dynamics, some input parameters are affected by same factors. For example, both pressure drop and heat transfer coefficient are affected by fluid condition like flow, vorticity and so on. Therefore each uncertainty of input parameters must have relationship. In the present research, scaling laws are used as input parameters in BEPU analysis to describe relations of each input parameters' uncertainties. Figure 3 is the image of uncertainty in BEPU analysis. This figure shows the relation of Pe number and Nu number of liquid metal. Pe number is horizontal axis, and Nu number is vertical axis. The width of lengthwise direction is the uncertainty in previous research method, or normal BEPU analysis (described in blue line). However Pe number, it is described in horizontal axis, also has an uncertainty because Pe number also depends on the condition of fluid, in the same way as Nu number. Thus we must consider the uncertainty in horizontal direction as well as the uncertainty in vertical direction (described in red line). By using scaling laws as input parameters, BEPU analysis can show more realistic result.

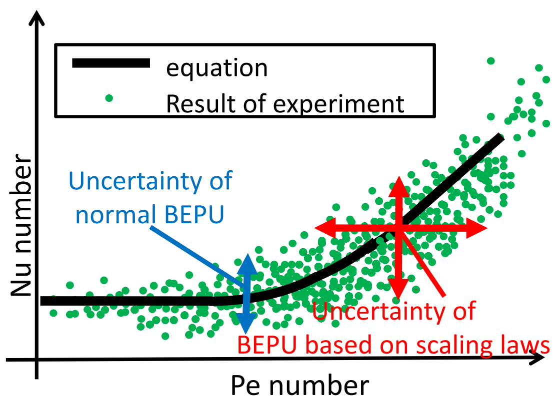

Fig.3 Image of uncertainty in BEPU analysis

4. NUMERICAL INVESTIGATION

4.1 Natural circulation decay heat removal in 4S reactor

A typical system of loop type 4S reactor is shown in Figure 4. The system consists of the reactor core, the primary heat transport system and the secondary heat transport system. Liquid sodium is used as both primary and secondary coolant. Intermediate Heat Exchanger (IHX) is installed to transport energy generated at the core from the primary system to the secondary system. Secondary coolant transports the energy, and heat energy is exchanged in Steam Generator (SG) by water. 4S reactor has two unique decay heat removal systems. One is Intermediate Reactor Auxiliary Cooling System (IRACS). IRACS is present in secondary loop, and it removes heat energy by using air flow when an accident occurs. The other is Reactor Vessel Auxiliary Cooling System (RVACS). RVACS is the cooling system which uses natural air flow as coolant. And it removes heat energy from the reactor, through the Reactor Vessel and Guard Vessel. From the view point of safety, there is no active system in RVACS and IRACS.

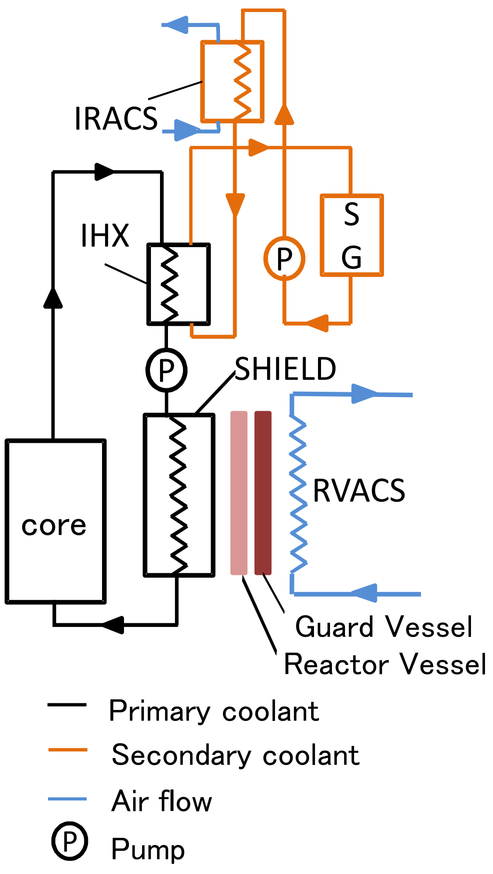

Fig.4 Model flow of 4S reactor

Figure 5 shows typical transients of the coolant temperature at core outlet in a long period of time. In the numerical simulation, third maximum coolant temperature (3rd peak temperature, 460°C) of core outlet is appeared in about 36 hours after the accident is happened. In this research, 4S simulation code made by Yamada (K. YAMADA (2011)) is used.

.

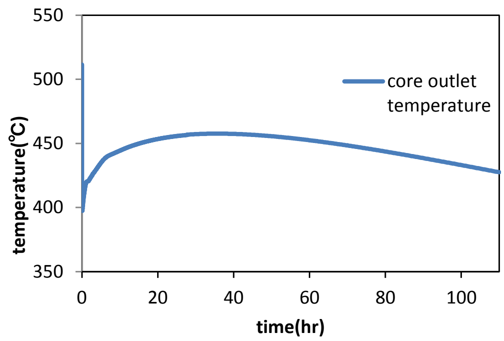

Fig.5 Typical transient coolant temperature (Core outlet)

4.2 Input variables and statistic properties

In the present research, dimensionless numbers are used as input parameters that are added uncertainties. Considered from previous research that is written by T. TAKATA (T.TAKATA, et al, (2003)), 14 parameters are considered in the present study. The parameters and their statistical properties are summarized in Table 1. PDF means probability density function.

Table 1
Input variables and statistic properties (BEPU based on scaling laws)

Input variable	PDF	coefficient of variance(%)
RVACS Re number	Normal	20
RVACS Pr number	Normal	10
RVACS Nu number	Normal	10
RVACS friction coefficient	Normal	30
RVACS inlet shape factor	Normal	10
RVACS outlet shape factor	Normal	10
CORE Re number	Normal	20
CORE decay heat	Normal	5
CORE friction coefficient	Normal	30
IRACS Re number	Normal	20
IRACS Pr number	Normal	10
IRACS Gr number	Normal	20
IRACS Nu number	Normal	10
IRACS friction coefficient	Normal	30

BEPU analysis which is based on scaling laws has never been researched yet. Therefore, it is also necessary to analyze normal BEPU analysis as a comparative analysis. The parameters and their statistical properties of normal BEPU analysis are summarized in Table 2.

Table 2
Input variables and statistic properties
(Normal BEPU : comparison analysis)

Input variable	PDF	coefficient of variance(%)
RVACS pressure drop	Normal	35
RVACS heat transfer coef	Normal	17
CORE pressure drop	Normal	35
IRACS pressure drop	Normal	37
IRACS heat transfer coef	Normal	11
decay heat power	Normal	5

Coefficient of variance in table 1 and 2 is described as equation (12).

$$\varepsilon = \frac{\sigma}{\mu} \tag{12}$$

ε is a coefficient of variance, μ is an average, and σ is a standard deviation. Quantities of dimensionless numbers are affected by the condition of coolant flow, coolant temperature and so on. Therefore variance and average of dimensionless number also depends on the condition. Hence in the present research, coefficient of variance is used to evaluate each uncertainty of input parameter.
Coefficient of variance in normal BEPU analysis is calculated from the result of each parameter's uncertainties in BEPU analysis based on scaling laws. For example, coefficient of variance of RVACS's pressure drop in normal BEPU analysis is calculated from the variance of RVACS's pressure drop in BEPU analysis based on scaling laws.
The quantities of parameters are 14 in BEPU analysis based on scaling laws, and 6 in normal BEPU analysis. Both analyses have 10 strata and 40 replicas, so 400 samples are calculated in the present research.

4.3 Calculation result

Figure 6 is the correlation ratio about the output result (3rd peak temperature of core outlet). This figure shows that the most important factor that affects the output result is decay heat, and the second most important parameter is Nu number of RVACS. This figure also shows that parameters of IRACS are less important than parameters of RVACS.
To compare normal BEPU analysis and BEPU analysis based on scaling laws, normal BEPU analysis is simulated. Correlation ratio about output results in normal BEPU analysis is shown in Figure 7. This result shows that the uncertainty of decay heat is not related to the uncertainty of output result. It is quite different from the result of BEPU analysis based on scaling laws.

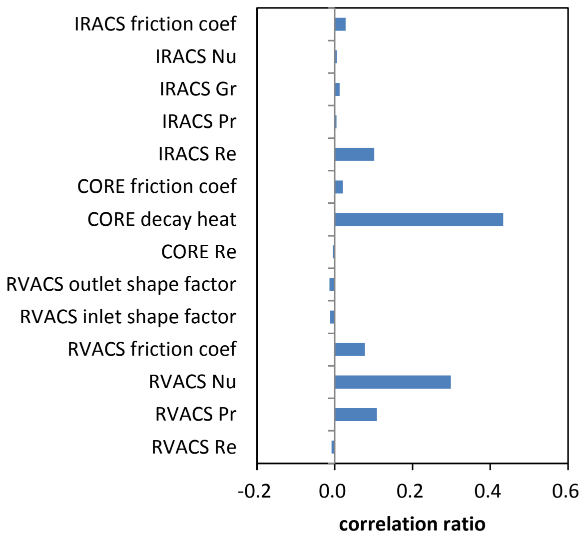

Fig.6 Correlation ratio of input parameters (BEPU based on scaling laws)

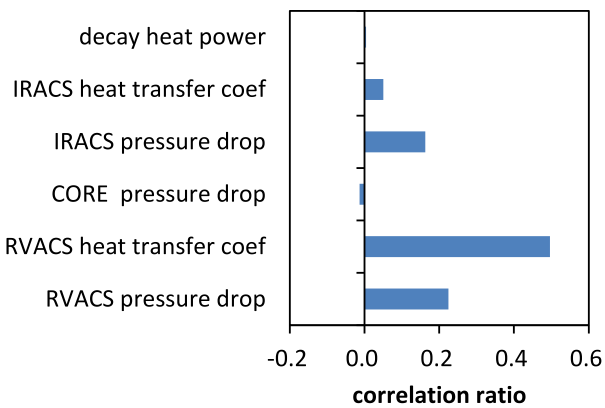

Fig.7 Correlation ratio of input parameters (normal BEPU)

This difference is caused by common effect in BEPU analysis based on scaling laws. For example, if uncertainties are added to Reynolds number of RVACS, uncertainty affects both pressure drop and heat transfer coefficient because equations which evaluate both parameters are represented by using Reynolds number. Therefore in BEPU analysis based on scaling laws, correlations are appeared between uncertainty of pressure drop and uncertainty of heat transfer coefficient. Figure 8 is the plot of the uncertainties of pressure drop and heat transfer coefficient in each sample in BEPU analysis based on scaling laws. ⊿p means the pressure drop of each samples and ⊿p(ave) means the average of pressure drop. Nu means the Nu number of each sample and Nu(ave) means the average of Nu number. And Figure 9 shows the plot in normal BEPU analysis. Figure 8 and figure 9 shows the cause of correlation between the uncertainty of heat transfer coefficient and the uncertainty of pressure drop.

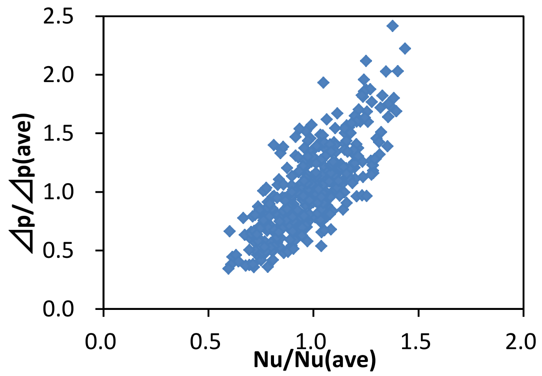

Fig.8 Variance of pressure drop and heat transfer coefficient in each sample (BEPU based on scaling laws)

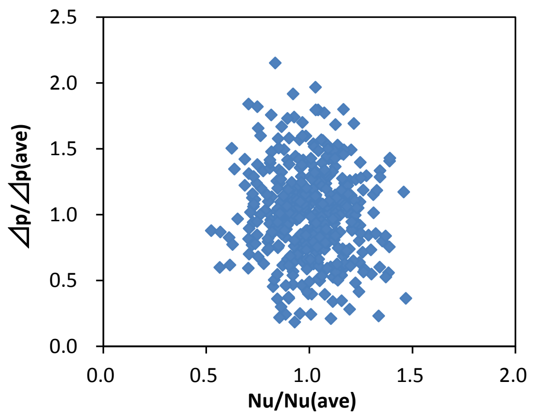

Fig.9 Variance of pressure drop and heat transfer coefficient in each sample (normal BEPU)

In BEPU analysis based on scaling laws, if the pressure drop of RVACS becomes higher, the peak temperature also becomes higher. And if the heat transfer coefficient of RVACS becomes higher, the peak temperature becomes lower. So this result means that in RVACS, the uncertainty of pressure drop and the uncertainty of heat transfer coefficient has positive correlation. On the other hand, Figure 9 shows that the result of normal BEPU analysis doesn't have such feature. So it is considered that such difference generates the difference about the features of correlation ratio between normal BEPU and BEPU analysis based on scaling laws. Such common effect, the relation of the uncertainty of pressure drop and the uncertainty of heat transfer coefficient, affects not only the correlation ratio but also the variance of output results. Table 3 shows the variance of output result (3rd peak temperature). In the case of BEPU analysis based on scaling laws, variance of output result in BEPU analysis based on scaling laws is less than the variance of output result in normal BEPU because the uncertainty of heat transfer coefficient is related to the uncertainty of pressure drop in BEPU analysis based on scaling laws.

Table 3
Variance and confidence level of 3rd peak temperature

		BEPU based on scaling laws	normal BEPU
variance(σ^2)		128.3	193.0
95% confidence level	upper limit(℃)	482.3	484.8

BEPU analysis based on scaling laws is the method to evaluate the uncertainty of output result from phenomenological theory. On the other hand, normal BEPU is the method to evaluate the uncertainty of output result from engineering judgment. The difference of evaluation makes the difference that BEPU analysis based on scaling laws can reduce the uncertainty of output result because this analysis method is more suitable for actual phenomena than normal BEPU analysis. And therefore BEPU analysis based on scaling laws can get rid of extra uncertainty which is resulting from engineering judgment.

5. CONCLUSION

To evaluate the uncertainty of 3rd peak temperature of 4S reactor in natural circulation decay heat removal phenomenon, BEPU analysis is used in the present research. And uncertainty which is obtained in BEPU analysis are estimated by using scaling laws, from a view point of phenomenological theory. To estimate the relation between the variances of dimensionless numbers and the variance of 3rd peak temperature of core outlet, Latin Hypercube Samplings and correlation ratio are used.
Calculation result indicates that the feature of correlation ratio in BEPU analysis based on scaling laws is quite different from the features of correlation ratio in normal BEPU analysis. Such differences are

caused by common effect, which means the relationship between the variance of pressure drop and the variance of heat transfer coefficient. This common effect reduces the variance of 3rd peak temperature of core outlet, and it makes the different result of each analysis’s correlation ratio. Judging from these results, it is important to think what parameters are chosen as input variables in BEPU analysis. And this research shows that BEPU analysis based on scaling laws can reduce the variance of output result because BEPU analysis based on scaling laws is chosen the input variables from a view point of phenomenological theory, not engineering judgment.

6. REFERENCES

Yamada, K. et al. (2012). A Yamaguchi, *Uncertainty of dimensionless number on BEPU analysis in 4S reactor,* AESJ 2012 spring meeting / G43, March(2012)

Mckay, M. D. (1995). *Evaluating Prediction Uncertainty*, NUREG /CR-6311 LA-12915-MS (1995).

Ueda, N. et al. (2003). ”Passive Reactor Dynamics and Load Following Characterristics of Sodium Cooled Super-Safe, Small and Simple Reactor,” *Proc. of ICONE-11, Icone-11-36539(2003)*

Takata, T. et al. (2010). *Stochastic Analysis of Natural Circulation Decay Heat Removal of Sodium Cooled Fast Reactor based on Latin Hypercube Sampling*, PSAM10 (16-2), June(2010)