Quantification of event sequence chronology using a continuous MCMC method for level 2 PSA of a nuclear power plant

Shogo Yabuuchi*, Takashi Takata, and Akira Yamaguchi

*Osaka University, Osaka, Japan

Abstract:

In this paper, we propose a new approach, continuous Markov Chain Monte Carlo (MCMC) method for the Probabilistic Safety Assessment (PSA). In the level 2 PSA, various phenomena and components response are modeled as headings of the event tree. However, the progression of the accident sequences is indefinite for the radioactive release and a structural failure may influence the subsequent scenario. Therefore, the sequences dealt with in the level 2 PSA is difficult to be quantified with the generally-used methods such as event tree/fault tree method.

As an example of the present approach, we first introduce a five-components system. And then we apply the method to the protected loss of heat sink (PLOHS) accident of the sodium-cooled reactor in which the system temperature increases gradually in time. The headings of the PLOHS accident are time-dependent and mutually dependent, and the order of occurrence is indefinite. It is found that the continuous MCMC method is easily applicable to complex accident sequences with dependencies and indefiniteness.

Keywords: MCMC, Level 2 PSA, PLOHS, ET, FT

1. INTRODUCTION

Today, there are many methods for Probabilistic Safety Assessment (PSA) of a nuclear power plant (NPP). The priority problems for NPP are radionuclide release scenarios in the event that the reactor core has been damaged. For assessment, the accident sequences are preliminarily prescribed as an event tree (ET) in which important key phenomena and critical events are defined as headings of the ET branches. However, the accident sequence is uncertain and time-dependent in nature. Therefore, one can not determine the order of the headings and quantify the branch probability.

In the present study, a new methodology based on a continuous Markov Chain Monte Carlo (MCMC) method is proposed to quantify an uncertain and time-dependent accident scenario that cannot be modeled only by the ET method. It has been applied to the protected loss of heat sink (PLOHS) accident of the sodium-cooled reactor in which the system temperature increases gradually in time. If it exceeds the limiting temperature of the heat transport system, the coolant boundary fails and the coolant spills out. This may cause deterioration of the cooling capability and result in the reactor core damage.

In the continuous MCMC method, subsequent plant states follow the current state. The uncertain and time-dependent scenario can be quantified by preparing a transition probability table of system states. The continuous MCMC method evaluates the system state probability sequentially and the ET does not need to be established in advance. Furthermore, external operations such as the recovery action can be easily considered. If we have thermal-hydraulic and/or mechanical system response models, they can be easily employed to calculate the event probability such as of a failure of structure and component by excessive thermal load. So, the present methodology can be applied to the quantification of complicated accident sequences.

* yabuuchi_s@qe.see.eng.osaka-u.ac.jp
2. METHODOLOGY OF CONTINUOUS MCMC METHOD

In the present study, a new methodology based on a continuous Markov Chain Monte Carlo (MCMC) method is proposed to quantify an uncertain and time-dependent accident scenario that cannot be modelled by the ET method. In the Markov process, subsequent plant states regarding time follow only the current state. It is said the Markov Chain. The Monte Carlo method statistically assesses the random phenomena occurrence using random number generation technique. Thus the order of occurrence of multiple events is decided by the process, which is one of random samples of population. Accordingly an uncertain and time-dependent accident scenario can be quantified by using the MCMC method in a statistical manner. For the assessment we must prepare a transition probability table of system states (see Fig 1). For assessing more complicated systems using the continuous MCMC method we sequentially evaluate system state probability by every time step. By using the continuous MCMC method, the ET does not need to be established in advance. Furthermore, external operations such as the recovery action can be easily considered. If we have thermal-hydraulic and/or mechanical system response models, they can be easily employed to calculate the event probability such as of a failure of structure and component due to an excessive thermal load. As above, the present methodology can be applied to the quantification of complicated accident sequences. In what follows, we explain points of the model construction by using the continuous MCMC method.

![Figure 1: The continuous Markov Chain Monte Carlo method](image-url)

2.1. Definition of system states

A system states is defined as combinations of headings in a PSA model. The system may have several components and failure modes depending on the number of required safety functions and the system complexity. It is necessary to take all states into consideration. Even if a state has very low appearance probability, it is to be defined to complete the whole system response model. An accident or event sequence is expressed as the combination of system states, which is called headings in the following. The sequences are quantified using the Monte Carlo method. The number of headings determines the number of size of the system model. For example, when the number of headings is \( n \) and the each heading has two alternatives; success or fault, the number of system states becomes two to the \( n \)th power.

2.2. Transition probability table

We must identify the success or failure probability of each heading. We identify them by preparing a transition probability table among system states. It is based on the evidence and the engineering judgment. It is important to consider the mutual dependency of the components. The transition of the system state is automatically determined by the Monte Carlo method at every time increment and there is no need preliminarily to construct the ET.

2.3. Time-step size and sample number

When we take a smaller time step, the execution time will increase though an evaluation will be more accurate. However, the order of magnitude of the system reliability does not change even if the time step is enlarged to some degree. So, if we change the time step, it is a tradeoff of the execution time
and the solution accuracy. Similarly, the number of samples should be selected appropriately in terms of computational efficiency and accuracy. (Ref [1])

3. EXAMPLE ANALYSES

We evaluated a simple system by using continuous MCMC method. This system contains components that failure is uncertain and time-dependent. It is a common situation to the Level 2 PSA.

3.1. Five-components system model

Here we consider a system model made of five components, “a”, “b”, “c”, “d”, and “e”. The system failure is defined as the triple failure of components “a”, “b” and “c”. In the initial condition, the system works successfully. The failure probability of a component is random and the failure of a component is influential to other component failure probabilities. In other words, failure probability of a component changes as a result of a failure of another component. Figure 2 shows the relationship of the five components’ failures. The components are designated as a, b, c, d, and e. The failure rate (failure probability per unit time) of “a”, “b”, “c”, “d”, and “e” are assumed to be 0.12, 0.06, 0.1, 0.2, and 0.4 respectively.

The dependency among the component failures are assumed as follows. If the component “a” fails, the failure rate of component “b” is doubled and vice versa. It is a situation that the failure of components “a” or “b” deteriorates the component working environment and failure rate is augmented. If both of components “a” and “b” fail, it is assumed that the failure rate of “c” is halved. The failure of component “d” influences the operation of component “a” significantly and it triples the failure rate of component “d”. Lastly, the failure rate of component “e” is tripled by the failure of component “b”.

The failure probability for each unit time \( \lambda_k(t)dt \) is shown as below by using the failure rate \( \lambda_k(t) \).

\[
\lambda_k(t)dt = \lambda_{0k}(1 - \exp(-m_k t))dt
\]

where the subscript \( k \) denotes the component. \( \lambda_k(t) \) increases in time and plateaus at \( \lambda_{0k} \).

![Figure 2: Five-components system](image)

3.2. Failure probability of the five-components system

The assessment condition was for time step of 0.01, sample number of 100,000. The system state probability, that is a triple failure of components “a”, “b” and “c”, is shown in Fig 3. It is noted that the system state probability is easily calculated using the continuous MCMC method although the system failure definition and the components failure dependency is complicated. It can be said, the proposed method is very useful for dynamic reliability analysis of a complex system model with dependency among the components. A typical example of such a sequence treated in the level 2 PSA. In the situation, the phenomena are time-dependent. One boundary failure influences another
failure. For example, if a pipe structural failure takes place, the internal pressure is released and other structural components become more resistant.

![Figure 3: Five-component system state probability with time](image)

4. QUANTIFICATION OF LEVEL 2 PSA SCENARIO OF FAST REACTOR

The protected loss of heat sink (PLOHS) accident is one of the risk-dominant scenarios of the sodium-cooled fast reactor. Failure of the decay heat removal system after the successful reactor shutdown results in the system temperature increases gradually in time. If it exceeds the limiting temperature of structural integrity of the heat transport system, the coolant boundary failure is anticipated and the coolant leaks out. This may cause deterioration of the cooling capability and result in the reactor core damage.

![Figure 4: Leakage paths of the Fission Products](image)

4.1. PLOHS assessment model construction

The scenario is schematically expressed in Fig 4. Figure 4 shows major components of the fast reactor and two leakage paths of the Fission Products (FP). This accident has some aspects, time-dependence, mutual dependence, and uncertain accident sequence.

The network of assessment is indicated as Fig 5. The components are expressed by the nodes in Fig 5. Five nodes are considered, i.e., the reactor core (C), the primary heat transport system (P), containment vessel (V), the intermediate heat exchanger (I), and the secondary heat transport system (S). If one considers either intact state or failed state for the five components, the number of states becomes $2^5 = 32$. In the initial condition, the system works successfully. The failure probability of
each node is changed as the temperature rises after the PLOHS event. And the failure of the component is random and the failure of a component is influential to other component failure probabilities. Such additional constructs are defined in Conditional Probability Table (CPT).

Considering the standby failure, we can calculate the failure probability $P_j(t)$ that a system fails by time $t$ as follows:

$$P_j(t) = \int_0^t \lambda_j \exp(-\lambda_j x) dx$$

$$= (1 - \exp(-\lambda_j t))$$

(2)

where $\lambda$ is a failure rate and subscript $j$ denotes the component.

And then, the probability of the recovery action is assumed to be given by a constant value. Once the recovery action is taken, it is considered that the heat removal will restart and the rise of the temperature will stop and drop. We express the cumulative failure probability after the recovery action as below.

$$P_j'(t) = P_j(t_{ra}) \ast (1 - P_j(t - t_{ra}))$$

$$= (1 - \exp(-\lambda_j t_{ra})) \exp(-\lambda_j (t - t_{ra}))$$

(3)

$t_{ra}$ is the time heat removal source is restarted. So, the failure probability after the recovery action changes into the decreasing direction. This idea is same with Ref [2]. The different point is that Dynamic Bayesian Network (DBN) has difficulty to consider uncertain accident sequences. The reason is we can apply the DBN when sequences are predefined. On the point the continuous MCMC method is effective.

![Figure 5: The continuous MCMC network for PLOHS](image)

4.2. PLOHS assessment results

We assessed the PLOHS accident. The assessment condition was for time step of 0.001, sample number of 5,000,000, and mission time of 30 hours. Fig 6 and Fig 7 indicate the results. Fig 6 shows that the failure probability distribution of each component. We can know the core and the primary...
coolant boundary are dominant. And the containment vessel has the lowest failure probability. Fig 7 shows state probability of the dominant sequences and Release total. The CPV is the sequence that contains failure of the core, the primary heat transport system, and the containment vessel simultaneously and the CIS is the sequence that contains failure of the core, the intermediate heat exchanger, and the secondary heat transport system. The release total contains the sequences of either CPV or CIS at least. From this figure, we can know that state probability of the sequence of CPV is kept down by defense function of the containment vessel and the sequence of CIS is dominant for state probability of the release total. What is more, it may be important to restore these dominant components within five hours from the PLOHS happened. These results are potentially valuable as they provide distributions rather than point values.

Figure 6: Failure probability distribution of each component

Figure 7: State probability of the dominant sequences and Release total

5. CONCLUSION

In this study, we introduced the method of the PSA model construction by the continuous MCMC method. This model solved the problem that there are difficulties to assess with the conventionally-known methods, such as ET or FT. As an examples, we assessed the protected loss of heat sink (PLOHS) accident of the sodium-cooled reactor in which the system temperature increases gradually
in time. The PLOHS accident is one of the severe accidents in the nuclear power plant. This method provides detailed changes of state probabilities with time. So, this method is better than the other method on the point that this method can provide a lot of information for our knowledge. Furthermore, this method enables the evaluation of actual system which needs a complex model construction by considering subsequent plant states follow only the current state. If we have thermal-hydraulic and/or mechanical system response models, they can be easily employed to calculate the event probability such as a failure of structure and component by excessive thermal load. It can be said this method is a very effective technique for the Level 2 PSA events in the field of nuclear power plant.

There are some difficulties for the continuous MCMC method to be improved. One is the transition probability table. Based on the plant response analysis the transition probabilities are evaluated more accurately. It is expected to couple the continuous MCMC method and thermal-hydraulic and structural response computation. Another is computational time and accuracy of the Monte Carlo method. When we introduce more accurate and realistic modeling in the MCMC method require considerable computational cost. It is a matter of tradeoff and would be appropriately judged according to the problem under consideration.

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References
