

Updating uncertainty of rare event occurrence probability based on Bayesian approach

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Abstract: Bayesian method and information criteria (information entropy and logarithmic likelihood) are applied to the seismic fragility evaluation and updating process. Those are informative and effective in the fragility update and the uncertainty reduction. Expected information entropy with respect to our belief on the seismic capacity is a useful figure-of-merit for estimating the importance or usefulness of performing a seismic qualification test before the results are obtained. Expected logarithmic likelihood is an appropriate measure and can be used to estimate the importance of the evidence which we have already known. Furthermore, it is meaningful that the expected entropy and logarithmic likelihood be weighted by the seismic hazard curves. It tells us the most effective way of the fragility update because the most risk-contributive PGA level depends on the subjective judgment on the hazard. It is recommended to perform additional vibration test is to be designed according to the expected entropy which suggests a reasonable guideline for the fragility update. The most contributive part of the fragility curve to the seismic failure frequency is the fragility tail in most cases and a test at the tail PGA level is more useful.

Keywords: Bayesian update, seismic fragility, information entropy, logarithmic likelihood

1. INTRODUCTION

Whole the spectrum of seismic-induced scenario is taken into consideration in the seismic probabilistic safety assessment (SPSA) of a nuclear power plant (NPP). Therefore, seismic fragilities need to be prepared for every failure mode of all the safety-related equipment and structures considered in the SPSA in order to quantify the residual seismic risk. Generally, a large number of systems and equipment are involved in the SPSA model. A problem is that our knowledge and data necessary for the fragility evaluation are not always sufficient and it is a difficult task to prepare the complete seismic fragility dataset specific to the NPP under consideration.

Preliminary evaluation of the seismic risk is performed using generic seismic fragilities[1] first of all and risk-dominant sequence and related systems and equipment are identified. According to the preliminary results, seismic responses and fragilities are refined with regard to the components in the dominant sequences using plant specific design and safety information. If it is found in this process that the seismic behavior is not well-understood or the uncertainty is large, additional seismic qualification tests would be performed or past seismic experience would be referred to update the seismic fragilities and to reduce the uncertainty.

The risk-dominant acceleration level is usually far beyond the design basis level. Recently, a series of seismic capacity test[2] was performed by Japan Nuclear Energy Safety Organization (JNES) in which the vibration table was used at higher acceleration than the design basis level. We have experienced earthquakes beyond the design earthquake level during these years in Japan. Such an example is the Tyuetsu-Oki earthquake[3] that occurred in July 2007 near Kashiwazaki-Kariwa nuclear power plant island. The information, i.e. the seismic capacity tests and the seismic experience, would be valuable and effective for the seismic fragility update if it is utilized appropriately. When we update the seismic fragility, the seismic experience should be selected, and an additional qualification test should be designed so that the fragility update using the information is useful and cost-effective. Therefore, we need a mathematical methodology for the seismic fragility update and a figure-of-merit for evaluating the importance of obtaining additional information.

Now we may raise a couple of question concerning the fragility update: (1) How the seismic experience

and seismic test data are used to update seismic fragilities? The method and the mathematical process in the fragility parameter estimate should be practical and transparent. (2) What the acceleration level should be of a seismic qualification test? How many samples should be tested to reduce the uncertainty cost-effectively to the level we want to know? Is available seismic experience or information useful enough to deserve our attention and efforts? What the figure-of-merit should be to measure the usefulness of the information? (3) Is the importance or the value of information the same for different analysts and NPP sites? A pessimistic analyst and an optimistic analyst would have different opinions on the same information. A fact that a component is damaged at an acceleration level would result in different feedbacks to a NPP in a high seismic zone and a NPP in a low seismic zone. Is there a universal methodology commonly applicable to evaluate the importance of the information?

The present study is directed to the updating of the seismic fragilities. In section 2, the seismic fragility model is described and features of the uncertainty in the fragility are discussed. The author proposes a Bayesian approach[4] that can reduce the uncertainty in the seismic capacity evaluation. The Bayesian approach for the fragility update[5] is described in section 3 to answer to the first question. The authors show the effectiveness of the present method and how the seismic risk can be revised by the Bayesian approach quantitatively. The information criterion which is well-known in the statistic science has been applied to the seismic fragility evaluation in relation to the second question and is explained in section 4.1. Also the consideration of the subjective belief on the seismic capacity and the seismic hazard are explained in section 4.2 and 4.3 in relation to the third question. Numerical examples are given to show the effectiveness of the Bayesian method and the information criterion.

2. Seismic Fragility

The seismic fragility is defined as a failure probability on condition that an earthquake occurs. The failure probability of equipment and structure under earthquake situation is often modeled using a double lognormal distribution. Hence the statistical parameters are the logarithmic mean and the logarithmic standard deviations concerning randomness and uncertainty.

A lognormal distribution is commonly used to describe a seismic fragility curve. With the peak ground acceleration (PGA) α as the intensity parameter of an earthquake, the fragility $F(\alpha)$ is expressed as:

$$F(\alpha) = \Phi \left[\frac{\ln(\alpha/C)}{\beta_R} \right] \text{ or } \frac{F(\alpha)}{d\alpha} = \frac{1}{\sqrt{2\pi}\beta_R\alpha} \exp \left[-\frac{1}{2} \left\{ \frac{\ln(\alpha/C)}{\beta_R} \right\}^2 \right] \quad (1)$$

where β_R and C are the variability of the fragility associated with the randomness and the seismic capacity of the equipment, respectively. $\Phi(\cdot)$ is the cumulative standard normal distribution function. Parameters of the function are the seismic capacity C and the logarithmic standard deviation β_R . The seismic capacity corresponds to the PGA level at which the seismic fragility or seismic-induced failure probability equals 0.5. The slope of the seismic failure probability curve corresponds to the magnitude of randomness β_R . If β_R equals zero, the failure behavior is deterministic. In other words, the equipment fails definitely beyond the PGA level C , and never fails below C .

It is noted that there is uncertainty in the estimate of the seismic capacity C . The PDF of the seismic capacity is also assumed to be lognormal:

$$f(C) = \frac{1}{\sqrt{2\pi}\beta_U C} \exp \left[-\frac{1}{2} \left\{ \frac{\ln(C/A_m)}{\beta_U} \right\}^2 \right]. \quad (2)$$

Here, A_m and β_U are the median value (or logarithmic mean) and logarithmic standard deviation of the seismic capacity, respectively. The variability is attributed to the inadequateness or imperfectness of our knowledge concerning the seismic capacity. It is easily understood that the failure probability curve corresponds to a certain value of C with a confidence level.

Comparing Eq. (1) and Eq. (2), we can interpret that β_R is the variability of the normalized seismic input level by the seismic capacity, i.e., α/C . On the other hand, β_U is the variability of the seismic capacity normalized by the median capacity, i.e. C/A_m . If we consider various confidence levels of the seismic

capacity, we can draw many fragility curves. Figure 1 shows the median (50% confidence level), 5% confidence level, and 95% confidence level fragility curves as a function of the PGA.

The shape of the fragility curve seems to be reasonable and consistent with the seismically-induced failure behavior appropriately. However, it is noted that there is no justified reason that one employs the distribution shape like that for the seismic failure probability of the equipment and structure. One of the drawbacks of the lognormal model is that one puts little emphasis on the fragility tail shape. Most of equipment and structures in the NPP has large seismic margins in general and the NPP is constructed in less hazardous site and built on a stiff bed-rock. Therefore, the lower tail of the fragility curves is most contributive to the seismic risk and of greater importance.

In the SPSA, the acceleration level which is most contributive to the seismic failure probability should be emphasized rather than the distribution shape itself. At the same time, the simplicity of the fragility function is very attractive and a significant advantage in the practical viewpoint of performing the seismic PSA. Therefore, it is reasonable that one utilizes the double lognormal fragility model and the parameters be adjusted to express the most contributive fragility tail. Thus we may consider an additional test or employing seismic experience to update the seismic fragility and to reduce uncertainty in the fragility tail.

3. BAYESIAN UPDATE

3.1. Bayesian Update Procedure

One can update the seismic fragility by performing shaking table tests, structural response analyses with advanced method or utilizing seismic experience. It is noted that the shaking table test is costly. The detailed analysis using high performance computer is also expensive as well. Thus we have to decide the most cost-effective test or analytical conditions based on the engineering judgment.

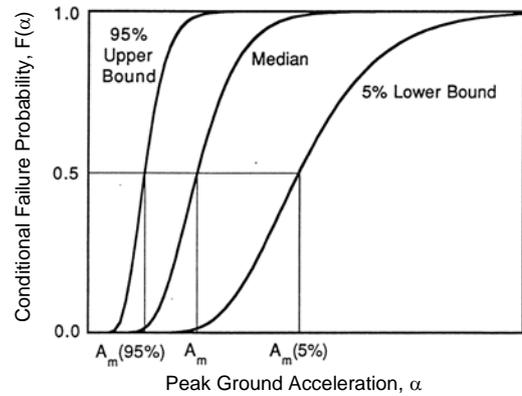


Fig. 1 Seismic fragility curves (median, 5% and 95% boundings with median of A_m).

Our knowledge on the seismic-induced failure is imperfect and its uncertainty is expressed by a PDF as in Eq. (2). When we have an empirical evidence E , it is reasonable that we update the function based on the new information. Let $L(E|C)$ be the likelihood of the evidence E on condition that we have a prior PDF $f(C)$ for the seismic capacity C , the posterior PDF of the seismic capacity after we obtain the evidence E is expressed as:

$$f(C|E) = \frac{f(C)L(E|C)}{\int_0^{\infty} f(C)L(E|C)dC} \quad (3)$$

This is the Bayes theorem and the process is called ‘‘Bayesian update’’.

Let us consider a situation where a shaking table test is performed at acceleration level α . If an analyst believes the seismic capacity of the component is C , and the randomness is β_R , the failure probability is given by Eq. (1) on condition that the seismic capacity is C , i.e. $F(\alpha|C)$. The seismic capacity is an uncertain variable with the median capacity A_m and logarithmic standard deviation β_U as seen from Eq. (2). When N components are tested at the same acceleration level α , there are $N + 1$ possibilities. The likelihood L_k is the probability that k components fail out of N trials and is calculated by the binomial distribution:

$$L_k(\alpha, N|C) = \binom{N}{k} F(\alpha|C)^k \{1 - F(\alpha|C)\}^{N-k}. \quad (4)$$

Substituting Eqs. (2) and (4) into Eq. (3), one obtains a mathematical expression of the Bayesian update.

Let us consider a shaking table test performed at the median capacity level as an example. The prior fragility parameters are assumed to be $A_m = 2.0$, $\beta_R = 0.3$, and $\beta_U = 0.4$. In this case, $N = 1$ in Eq. (4) and we have two possibilities, i.e. success or failure. The likelihood function is either:

$$L_0(\alpha|C) = 1 - F(\alpha|C) \text{ (success), or} \quad (5)$$

$$L_1(\alpha|C) = F(\alpha|C) \text{ (failure)}. \quad (6)$$

Figure 2 shows the results of the Bayesian update of the seismic capacity. The seismic capacity level is normalized with respect to the median capacity, i.e. $\theta = C / A_m$ in Fig. 2. Thus the test level is $\theta = 1$. Two cases are considered in Fig. 2; the chained line shows the case in which the component failed; the solid line corresponds to the successful test with no failure. It is seen that the PDF of the seismic capacity is substantially changed according to the information obtained from a single component test at the median capacity PGA level.

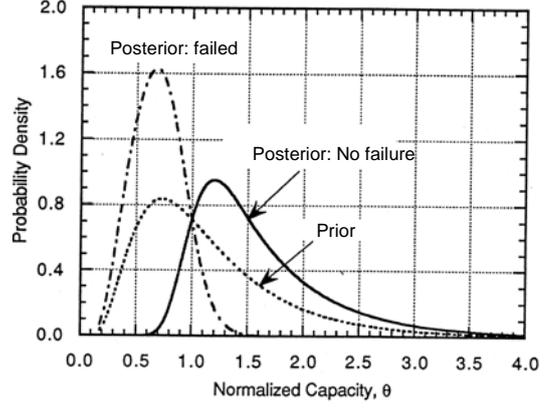


Fig. 2 Bayesian update: prior and posterior PDFs of the seismic capacity.

3.2. Parameterization of Posterior Seismic Capacity

In the seismic PSA of the nuclear power plants, a lognormal fragility model is commonly used because of simplicity and mathematical convenience. However, the posterior distribution is not always lognormal because the posterior distribution is a product of the prior distribution and the likelihood function as seen from Eq. (3). Therefore, the posterior distribution is to be fitted to the lognormal function. Since β_R reflects the randomness and cannot be updated, one needs to evaluate two parameters of Eq. (2), i.e. A_m and β_R .

When several seismic hazard curves, that is annual frequency that an earthquake acceleration level exceeds α , are considered with several opinions with different credibility, the annual frequency of failure of an equipment is expressed by a convolution integral as:

$$F_r = - \int_0^\infty \int_0^\infty \sum_{i=1}^M w_i \frac{dh_i(\alpha)}{d\alpha} F(\alpha|C) f(C|E) d\alpha dC. \quad (7)$$

where $h_i(\alpha)$ and w_i are the i -th hazard curve and its weight factor or relative credibility, respectively. M is the number of different hazard curves

We may have several methods to estimate the fragility parameters. One is the least square fitting to obtain the lognormal fragility parameters. Another is tail-oriented parameterization that uses the 95% lower bound and median values of the capacity for the parameterization. Alternatively, we may use discrete posterior PDF without resorting to parameterization of the lognormal distribution, i.e. direct computation of Eq. (3). It is clear that the third gives accurate result. We have compared the three methods in terms of the annual frequency of failure in Ref.[5], which shows the second method is applicable to the parameterization of the posterior fragility curve and the fragility curve tail is important in the SPSA. If we put emphasis on the shape of the fragility tail it should be nearly equivalent to the non-parametric method using the discrete posterior.

4. Information Criteria

4.1 Logarithmic Likelihood and Information Entropy

Here a question arises about the fragility qualification by a shaking table test. What acceleration level should be selected in the shaking table test? How many components should be tested? The logarithmic likelihood and the information entropy will give the answer to the questions. It is interesting to know the importance or worth of new information from the viewpoint of the fragility updating. The importance of a new evidence can be evaluated by the logarithmic likelihood V_k :

$$V_k(\alpha, N|C) = -\ln L_k(\alpha|C). \quad (8)$$

where L_k is the likelihood. The likelihood L_k is the probability that the evidence is observed and is given

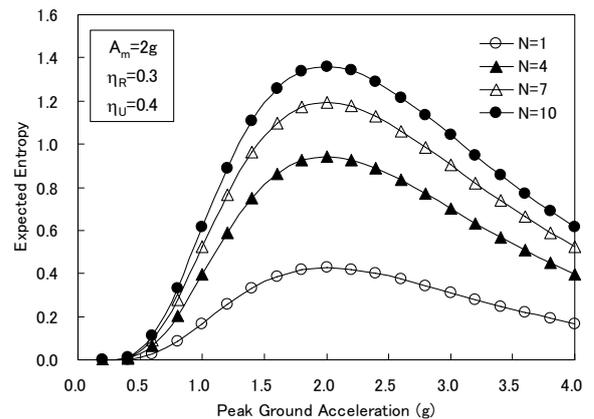


Fig. 3 Expected Entropy of a test as a function of test PGA level and number of tested components.

by Eq. (4) in the present case.

The logarithmic likelihood varies from zero to infinity. If the component is very rigid and seismic-resistant, the likelihood of component failure is nearly zero and the logarithmic likelihood is infinite. It implies that the failure of a very resistant component is a rare event. Viewing it from the other side, we understand that an occurrence of a rare event is very valuable and the importance of the information would be significant. On the other hand, the logarithmic likelihood is zero if the failure probability or the likelihood is unity. We take it granted for that a very fragile component should fail in the shaking table test and no noteworthy information is obtained from the test. Therefore, the logarithmic likelihood is zero which implies the test result is almost meaningless or the information value is negligibly small. It can be said that the logarithmic likelihood is a useful measure to judge the test result is notable or not after the test.

Since we do not know the test results in advance, the expected value of the logarithmic likelihood with respect to all the possibilities is a point of concern. It is defined as the information entropy, E . We assume the test result is either fail or success. The information entropy is defined as:

$$E(\alpha|C) = -F(\alpha|C) \ln F(\alpha|C) - \{1 - F(\alpha|C)\} \ln \{1 - F(\alpha|C)\}. \quad (9)$$

When N components are tested at the same acceleration level, there are $N + 1$ possibilities. Thus the general expression of the entropy is:

$$E(\alpha, N|C) = -\sum_{k=0}^N L_k \ln L_k. \quad (10)$$

The information entropy tells us how the fragility estimate is efficiently updated with the present information or evidence. In other words, additional test or analysis for the seismic fragility update should be designed so that the entropy becomes the maximum.

4.2 Logarithmic Likelihood and Information Entropy - Seismic capacity Averaged

The importance of the additional information on the seismic fragility update is influenced by the subjective judgment on the seismic capacity of the analyst since the logarithmic likelihood and the information entropy depends on the seismic capacity. Thus the importance is quantified by taking average of the logarithmic likelihood and the information entropy in terms of the seismic capacity PDF.

It is reasonable that the expected information entropy with respect to the seismic capacity should be utilized to evaluate the importance of the additional information such as a vibration test or seismic experience. The expected values of the logarithmic likelihood $\bar{V}_k(\alpha)$ and the information entropy $\bar{E}(\alpha)$ are given by:

$$\bar{V}_k(\alpha) = \int_0^\infty V_k(\alpha|C) f(C) dC = -\int_0^\infty f(C) \ln L_k(\alpha|C) dC, \quad \text{and} \quad (11)$$

$$\bar{E}(\alpha) = \int_0^\infty E(\alpha|C) f(C) dC = -\int_0^\infty f(C) \sum_{k=0}^N L_k(\alpha|C) \ln L_k(\alpha|C) dC. \quad (12)$$

The expected entropy is interpreted as the importance of a test at acceleration level α before the test is performed for the analyst who believes the seismic capacity PDF is $f(C)$. On the other hand, the expected logarithmic likelihood is the importance of the test result that k components failed at acceleration level α for the analyst who believes the seismic capacity PDF is $f(C)$.

Figure 3 shows the expected information entropy for the shaking table test of a component with the fragility parameters given in section 3.1. The horizontal axis indicates the acceleration level at which the test is performed. Four curves drawn in Fig. 6 indicate the number of tested components, that is $N=1, 4, 7$ and 10 . It is seen that the expected entropy reaches maximum at the median acceleration level at $2.0g$ regardless to the number of components. It is consistent with our intuition. If the test acceleration level is too low, we expect most of the tested components survive. On the other hand, many will fail if the acceleration level is too high. We see another important tendency from Fig. 3. As the number of tested components increases, the increment of the expected entropy is reduced. We may suggest that limited number (one or two) of components is tested at the median capacity level. Afterward, another acceleration level would be selected according to the updated PDF of the seismic capacity.

Figures 4 and 5 show the expected logarithmic likelihood. The number of tested components is four, that is $N=1, 4, 7$ and 10 . Figure 4 shows the results when all the components survive in the test. The expected

logarithmic likelihood for successful test results is monotonically increasing. The results come up to our expectation. It is easily understood that the observation is very significant if the components survive at higher acceleration level than the median capacity. At the acceleration level lower than 1.0g (half of the median capacity), the expected logarithmic likelihood is small. It is a matter of course that the seismic qualification test at the acceleration level considerably lower than the median capacity is almost meaningless from the fragility updating purpose. The expected value of the logarithmic likelihood when all the components failed in the test is shown in Fig. 5. This figure shows opposite tendency to the all component survival case. Fig. 4 and Fig. 5 are extreme cases that all components fail or all components survive, respectively.

The expected entropy is interpreted as the importance of a test at acceleration level α before the test is performed for the analyst who believes the seismic capacity PDF is $f(C)$. On the other hand, the expected logarithmic likelihood is the importance of the test result that k components failed at acceleration level α for the analyst who believes the seismic capacity PDF is $f(C)$. In the procedure, we can evaluate the importance of additional information for updating the seismic fragility. Here we verify how the information entropy works and seismic fragility is updated using shaking table tests results and the Bayesian method.

Let us consider a shaking table again at the median capacity level. If one component is tested and it survives, the expected logarithmic likelihood (importance of the observation) is calculated as 1.22 (see Fig. 4; $N=1$; $PGA=2.0g$). Now we consider the component test at lower acceleration than the median capacity level. To achieve the same value of the expected logarithmic likelihood, we should use four components at 1.282g and all of them survive. The former test (one component test at 2g acceleration level) is named Case 1 and the latter one (four components test at 1.282g) is Case2 in the following.

Table 2 shows the Bayesian update results using the test information. From the evidence that the components do not fail in the test, the median capacity increases and the uncertainty is reduced. The median capacity increment is larger in Case 1 (from 2.0 to 2.55) than in Case2 (from 2.0 to 2.42). The reason is that the test acceleration level is high in Case 1 and we are confident that the capacity is higher than we considered in advance of the test. On the other hand, the reduction of the uncertainty is larger in Case 2 (from 0.4 to 0.255) than in Case 1 (from 0.4 to 0.286). We are more confident that the failure probability at the test level (1.28g) should be smaller than we expected in the prior fragility because no failure occurs in the test. In other words, we have

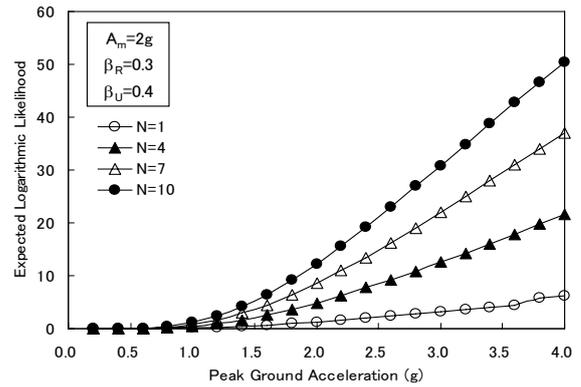


Fig. 4 Expected likelihood of a all the components survival in a shaking table test as a function of the test acceleration level and number of components.

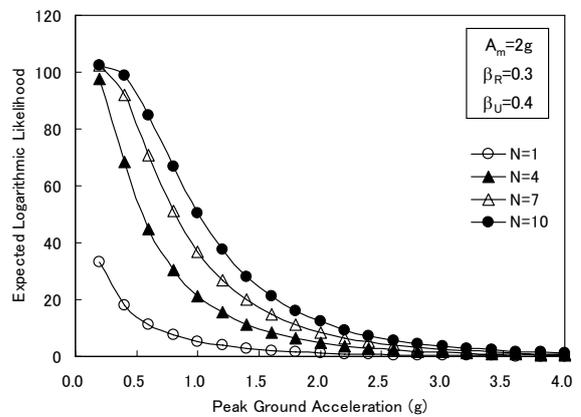


Fig. 5 Expected likelihood of all the components failure in a shaking table test as a function of the test acceleration level and number of components.

Table 2 Prior and posterior fragility parameters

	Prior	Posterior	
		Case 1 (one test at 2.0g)	Case 2 (four test at 1.28g)
A_m (g)	2.0	2.548	2.424
β_R	0.3	0.3	0.3
β_U	0.4	0.286	0.255

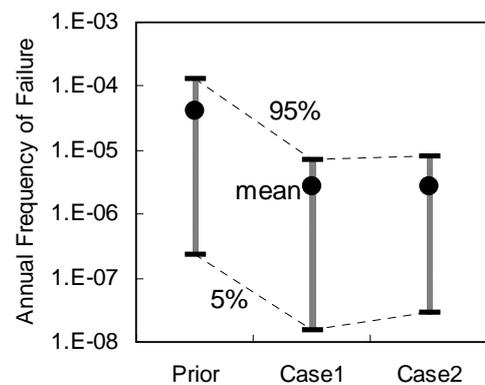


Fig. 6 Annual frequency of failure evaluated for prior fragility and posterior fragilities (Case1 and Case 2).

obtained enough evidence to decrease the failure probability at low PGA level. It is the reason that more reduction of the uncertainty is observed in Case 2 than in Case 1.

Here we evaluate the annual frequency of failure of the equipment using hazard curves. Now let us investigate how the two cases influence on the annual frequency of the component failure. The posterior fragility curves are convoluted with the hazard curves by Eq. (7) to obtain the annual frequency of failure. The results are shown in Fig. 6 with mean, 95% and 5% bounding values. It is noted that the mean value is the same (2.8E-6) for both posterior cases although the seismic test level and tested components are different.

4.3 Logarithmic Likelihood and Information Entropy - Hazard Averaged

It is reasonable to consider the importance of new evidence is different for the high seismic zone plant and low seismic zone plant. Even though evidence that a component survives a shaking table test at a certain acceleration level is useful information for a low seismic zone plant, it is not necessarily meaningful for a high seismic zone plant. It is because the risk-dominant acceleration level is depends on the seismicity of the plant. In other words, the shaking table test should be performed taking both of the plant seismic capacity and the plant seismicity into account.

$\bar{V}_k(\alpha)$ in Eq. (11) and $\bar{E}(\alpha)$ in Eq. (12) are the expression of expected logarithmic likelihood and expected entropy with respect to the seismic capacity. Here we multiply the quantities with the seismic hazard function and define $\tilde{V}_k(\alpha)$ and $\tilde{E}(\alpha)$:

$$\tilde{V}_k(\alpha) = -\int_0^\infty \omega_i \frac{dh_i(\alpha)}{d\alpha} V_k(\alpha|C) f(C) dC = -\omega_i \frac{dh_i(\alpha)}{d\alpha} \bar{V}_k(\alpha), \text{ and} \quad (13)$$

$$\tilde{E}(\alpha) = -\int_0^\infty \omega_i \frac{dh_i(\alpha)}{d\alpha} E(\alpha|C) f(C) dC = -\omega_i \frac{dh_i(\alpha)}{d\alpha} \bar{E}(\alpha). \quad (14)$$

Hazard-weighted expected entropy $\tilde{E}(\alpha)$ is the importance of a shaking test at acceleration level A for the analyst who believes the seismic capacity PDF is $f(C)$ and the seismic hazard is $h(\alpha)$. Hazard-weighted logarithmic likelihood $\tilde{V}_k(\alpha)$ is the importance of an evidence that k components failed at an earthquake or a shaking test of the acceleration level A for the analyst who believes the seismic capacity PDF is $f(C)$ and the seismic hazard is $h(\alpha)$.

Figure 7 shows an example of seismic hazard. There are eight hazard curves. Four of them shown by solid lines have 0.15 weight factor and the others have 0.1 credibility. Figure 8 shows the hazard-averaged expected entropy $\tilde{E}(\alpha)$. It is interesting that $\tilde{E}(\alpha)$ is maximized at 0.6g although the expected entropy $\bar{E}(\alpha)$ reaches the maximum at the median capacity level, i.e. 2.0g as seen from Fig. 3. It means the fact that the analyst believing the seismic hazard curve as in Fig. 7 considers a test at high PGA level beyond 1.5g will provide little information. It is because that the seismic hazard level for the plant is significantly low at the PGA level. If the plant is sited in the seismic zone which seismic hazard curves are given by Fig. 7, the additional seismic test should be performed the 0.6g, one third smaller than the case in which no seismic hazard consideration is given.

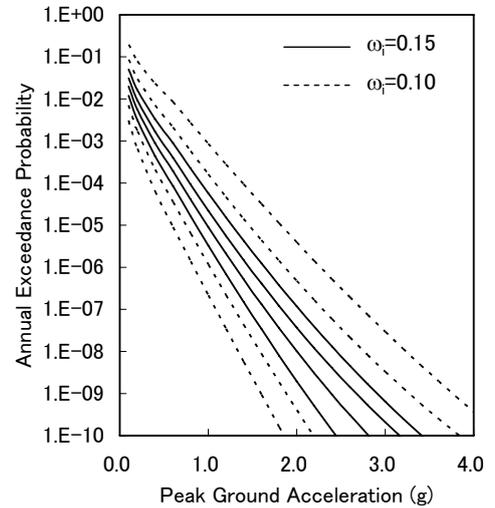


Fig. 7 Hazard curves.

The hazard-averaged expected logarithmic likelihood that all tested components survive ($k = 0$), $\tilde{V}_{k=0}(\alpha)$, is shown in Fig. 9. The expected likelihood (not hazard-averaged) shown in Fig. 4 is monotonically increasing as the PGA. It has the same tendency as the hazard-averaged expected entropy shown in Fig. 8, which has uni-peak shape. We may interpret the results as that the survival of components at high acceleration level is contrary to our expectations and is a rare event. Hence the logarithmic likelihood increases as the PGA and the survival in a high acceleration test is of great value and effective in the Bayesian update. On the other hand, Fig. 9 suggests that the update of the seismic fragility using the evidence has little contribution to the seismic risk because the seismic hazard is small enough at high PGA level. We are not interested in what we believe would not happen.

From this example, it is recommended to have a shaking table test at $PGA=0.6g$ for the analysts who believe seismic capacity $f(C)$ and seismic hazard $h(\alpha)$ given in Fig. 7. It is noted that the PGA level is smaller than the median capacity, $2.0g$. The test at the median capacity level is recommended as shown in the previous section for the analysts who believe seismic capacity $f(C)$ regardless of seismic hazard. When $\hat{V}_k(\alpha)$ is large enough for the analysts having seismic capacity $f(C)$ and seismic hazard $h(\alpha)$, the evidence that k components failed in the earthquake or shaking table test at acceleration α should be utilized for the Bayesian updating.

5. CONCLUSION

The Bayesian method and the information criteria are informative and effective in the fragility update and the uncertainty reduction. The posterior fragility curve is not lognormal function in the Bayesian update process. However, there is an advantage to use the lognormal fragility model because it is simple and mathematically convenient. It is recommended to fit the posterior seismic capacity PDF to a lognormal function and to obtain the parameters based on the most contributive PGA level of the fragility curve, i.e. the fragility tail.

Expected information entropy with respect to the seismic capacity is a useful figure-of-merit for estimating the importance or usefulness of performing a seismic qualification test (before the results are obtained). The expected logarithmic likelihood with respect to the seismic capacity is an appropriate measure and can be used to estimate the importance of evidence of which we already know the results. Additional vibration test is to be performed so that the expected entropy becomes the maximum. The procedure suggests a reasonable guideline for the fragility update by performing additional test. If we already have observations (test results or seismic experience) concerning the seismic fragility, we just need to calculate the expected logarithmic likelihood. If it is large enough, the observation should be used in the fragility update process. The entropy and the logarithmic likelihood should be averaged by the seismic capacity and seismic hazard curves. It tells us the most effective way of the fragility update because the most risk-contributive PGA level depends on the subjective judgment on both of the seismic capacity and hazard.

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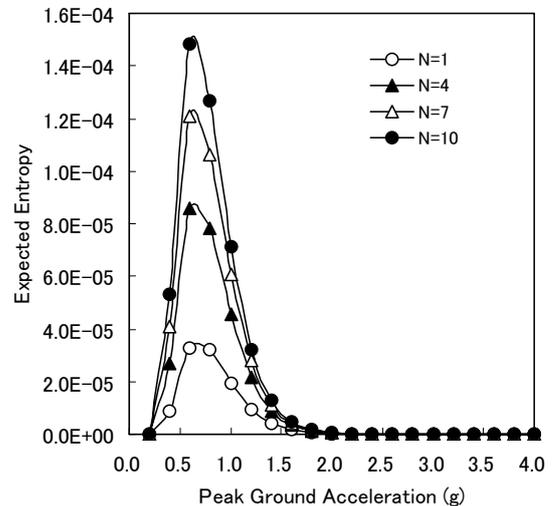


Fig. 8 Hazard-averaged expected entropy.

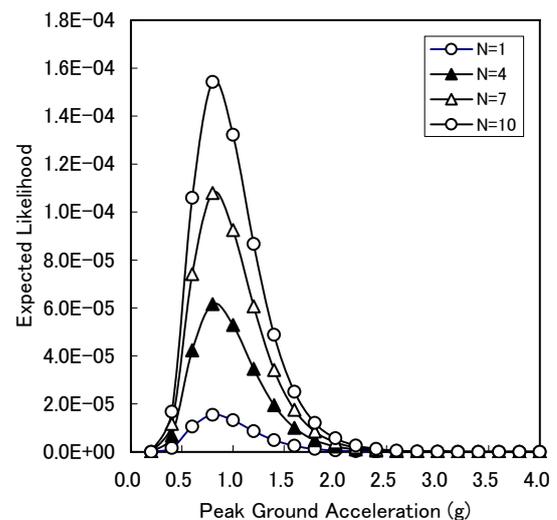


Fig. 9 Hazard-averaged expected likelihood